Annals of Optimization With Applications

www.anowa.reapress.com

Research Expansion Alliance

Ann. Optim. Appl. Vol. 2, No. 1 (2026) 91–98.

Paper Type: Original Article

Hybrid Metaheuristic Design for Solving the Cardinality-Constrained Portfolio Problem: Comparative Analysis in Six Equity Markets

Mohammad Hossain Noripour*

Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran; M.noripour79@gmail.com.

Citation:

Received: 09 August 2025 *Revised: 12 October 2025* *Accepted: 21 December 2025*	Noripour, M. H. (2026). Hybrid metaheuristic design for solving the cardinality-constrained portfolio problem: Comparative analysis in six equity markets. *Annals of Optimization With Applications*, 2(1), 91-98.

Abstract

We present a hybrid metaheuristic for mean–variance portfolio optimization with a cardinality constraint. The algorithm combines Continuous Ant Colony Optimization (CACO), a Genetic Algorithm (GA), and Artificial Bee Colony (ABC), with elitism and localized mutation to balance exploration and exploitation. Across six benchmark equity markets (S&P 100, FTSE 100, Hang Seng, Nikkei 225, DAX 100, XU100/XU030), the method yields lower mean/Variance Return Error (VRE) and smaller Mean Euclidean Distance (MEUCD) to the unconstrained efficient frontier than competing settings. On S&P 100, the best configuration achieves VRE ≈ 2.52, MRE ≈ 0.70, and MEUCD ≈ 1e-4. The approach remains competitive on more volatile markets (e.g., DAX, Nikkei), indicating robustness.

Keywords: Portfolio optimization, Cardinality constraint, Metaheuristics, Ant colony optimization, Genetic algorithms, Artificial bee colony.

1 | Introduction

Portfolio selection is a fundamental problem in financial decision-making, where the objective is to allocate capital among a set of assets to balance expected return and risk. In 1952, Markowitz [1] introduced the Mean–Variance (MV) model, which quantitatively expressed portfolio risk based on the covariance between assets. This seminal work, which earned a Nobel Prize in 1991, laid the foundation for modern portfolio theory [2].

However, the original MV formulation ignores many practical constraints such as transaction costs, minimum lot sizes, and cardinality restrictions. While the Unconstrained Portfolio Optimization (UCPO) problem can

Corresponding Author: M.noripour79@gmail.com
https://doi.org/10.48314/anowa.v2i1.69

be efficiently solved using quadratic programming, the Cardinality-Constrained Portfolio Optimization (CCPO) problem becomes NP-hard, rendering exact methods impractical for large-scale instances.

To address these challenges, numerous heuristic and metaheuristic approaches have been proposed. Cura [3] introduced a GA-based approach for CCPO, while Mozaffari et al. [4] and Baykasoğlu et al. [5] employed evolutionary and swarm-based methods with encouraging results.

More recently, new metaheuristic applications have been proposed: Bulani et al. [6] improved portfolio management by integrating clustering with Particle Swarm Optimization (PSO), Mwamba et al. [7] applied NSGA-III for multi-objective portfolio optimization, and Zarzycki [8] proposed a bacterial foraging algorithm combined with robust risk management. In addition, Erwin and Engelbrecht [9] developed a set-based PSO framework for portfolio optimization, and reviews such as Tomar et al. [10] highlight the growing importance of hybrid and swarm-intelligence approaches in finance. Despite these advances, most algorithms still face difficulties in maintaining diversity, balancing exploration and exploitation, and avoiding premature convergence.

In this study, we propose a novel hybrid metaheuristic that integrates Continuous Ant Colony Optimization (CACO), Genetic Algorithm (GA), and Artificial Bee Colony (ABC) into a unified optimization framework for solving the cardinality-constrained portfolio selection problem. Each component contributes complementary strengths: CACO provides an efficient probabilistic search mechanism with Gaussian-based solution construction, GA introduces elitism and crossover operators to preserve and propagate high-quality solutions across generations, and ABC contributes localized mutation strategies that help maintain diversity and prevent premature convergence. By combining these mechanisms, the algorithm achieves a balanced trade-off between global exploration of the solution space and local exploitation around promising portfolios.

The proposed hybrid algorithm is evaluated on six benchmark markets covering diverse financial environments. Its performance is measured using three error metrics relative to the unconstrained efficient frontier: Mean Return Error (MRE), Variance Return Error (VRE), and Mean Euclidean Distance (MEUCD). Results show that the method generates portfolios closer to the theoretical frontier, offering improved accuracy and robustness compared to state-of-the-art approaches.

The remainder of this paper is organized as follows. Section II presents the mathematical formulation of the problem. Section III describes the proposed hybrid algorithm. Section IV outlines the experimental setup and reports the results, and Section V concludes the paper with key findings and directions for future research.

2 | Problem Formulation

We adopt the Mean–Variance (MV) portfolio optimization model with an additional cardinality constraint to limit the number of assets in the portfolio. Let w_i denote the weight of asset i in the portfolio, μ_i its expected return, and Σ the covariance matrix of asset returns. The classical MV model seeks to minimize portfolio variance for a given target return or maximize return for a given risk level:

$$\text{Min } wt \sum wi. \tag{1}$$

$$\text{s.t. } \sum_{i=1}^{N} wi = 1. \tag{2}$$

$$wi \geq 0 \ \forall i. \tag{3}$$

$$\sum_{i=1}^{N} \mu i.\, wi \geq R \text{ target}. \tag{4}$$

To incorporate the cardinality constraint, we introduce a binary selection variable $Z_i \in \{0,1\}$ for each asset, where Z_i=1indicates inclusion of asset I in the portfolio and Z_i=0 otherwise. The constraint is expressed as:

$$\sum_{i=1}^{N} Zi \le K. \tag{5}$$

$$wi \le ui.zi \ \forall i. \tag{6}$$

Here, K is the maximum number of assets allowed in the portfolio, and uiu_iui is the upper bound on the weight of asset i. The binary–continuous linking constraint wi ≤ ui.zi ensures that weights are nonzero only for selected assets.

In this work, the quality of a constrained portfolio is evaluated relative to the unconstrained efficient frontier. Specifically, we use:

I. MRE: The average deviation of the portfolio's expected return from the frontier point with the same variance.

II. VRE: The average deviation of the portfolio's variance from the frontier point with the same return.

III. MEUCD: The average distance between the constrained solution and its corresponding point on the efficient frontier in risk–return space.

These measures provide a consistent basis for comparing constrained solutions with the idealized unconstrained benchmark across multiple markets.

Fig. 1 illustrates the geometric interpretation of these error measures. A constrained solution (x∗,y∗) is projected onto the unconstrained efficient frontier; the horizontal and vertical distances represent VRE and MRE, respectively, while the Euclidean distance captures the combined.

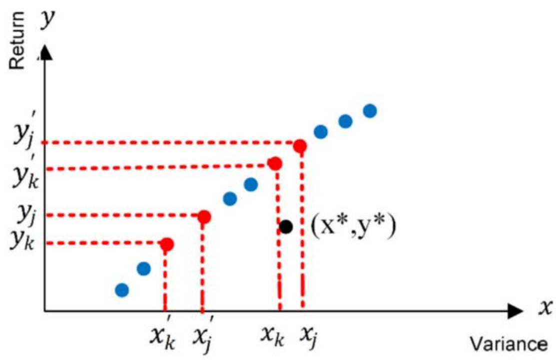

Fig. 1. Illustration of error measurement: deviation of a constrained portfolio.

3 | Proposed Hybrid Algorithm

The proposed approach integrates three metaheuristic methods, CACO, GA, and ABC, into a unified hybrid framework for solving the cardinality-constrained mean–variance portfolio optimization problem. The main objective is to exploit the complementary strengths of each algorithm:

I. CACO provides guided exploration of the solution space through pheromone-based probabilistic construction of portfolios.

II. GA facilitates recombination and mutation of high-quality solutions to generate diverse offspring.

III. ABC conducts a local search around promising candidates to refine portfolio weights and improve solution quality.

3.1 | Initialization

The algorithm begins by initializing pheromone trails in CACO, generating an initial population of feasible portfolios, and evaluating their objective function values. Elitism is applied to retain the top-performing solutions for subsequent iterations. Units

3.2 | CACO Solution Construction

In each iteration, a set of artificial ants constructs new portfolio weight vectors. Pheromone values guide the probability of selecting specific assets and allocating weights, while evaporation controls the balance between exploration and exploitation. Infeasible portfolios are repaired to meet budget, non-negativity, and cardinality constraints.

3.3 | Genetic Algorithm Operators

A subset of the best solutions undergoes GA operations: tournament selection chooses parent portfolios, simulated binary crossover generates offspring, and adaptive mutation perturbs weights or toggles asset inclusion. Mutation intensity is adjusted based on improvement trends, with higher mutation applied in stagnant regions of the search space.

3.4 | ABC Local Search

The ABC phase assigns employed bees to explore the neighborhoods of elite solutions, while onlooker bees are biased toward portfolios with higher fitness. Scout bees introduce new random solutions to prevent premature convergence.

3.5 | Fitness Evaluation and Constraint Handling

Portfolios are evaluated using one of the three performance metrics (MRE, VRE, or MEUCD) depending on the optimization target. Constraint-handling mechanisms include budget rescaling, elimination of small negative weights, and trimming the asset set to the best-scoring K assets.

3.6 | Pheromone Update and Elitism

At the end of each iteration, pheromone trails are updated based on the quality of constructed portfolios, with evaporation applied to reduce the influence of older solutions. Elitism ensures that the best solutions found so far are preserved for the next iteration.

The process repeats until a stopping criterion is met, such as a maximum number of iterations or a convergence threshold. The final output is the best feasible portfolio lying on the constrained efficient frontier.

4 | Experimental Setup

4.1 | Datasets

To evaluate the proposed hybrid algorithm, we conducted experiments on six benchmark equity markets widely used in portfolio optimization literature:

I. S&P 100 (USA)–a representative large-cap market from the New York Stock Exchange and NASDAQ.

II. FTSE 100 (UK)–the most liquid UK equity index, representing a diverse industrial and financial base.

III. DAX 100 (Germany)–a high-volatility European market with strong cyclical sectors.

IV. Hang Seng Index (Hong Kong)–characterized by exposure to both global and mainland China economic trends.

V. Nikkei 225 (Japan)–a mature but volatile market driven by export-oriented industries.

VI. Borsa Istanbul (XU100/XU030)–an emerging market with higher volatility and lower liquidity relative to developed indices.

For the global markets (S&P 100, FTSE 100, DAX 100, Hang Seng, Nikkei 225), we employed datasets from the OR-Library, which provide historical mean returns and covariance matrices calculated from multi-year daily price data. For the Turkish markets, historical daily closing prices were collected from official exchange data and processed to compute average returns and covariance matrices.

4.2 | Parameter Settings

The hybrid algorithm requires several parameters for each of its three components:

I. CACO parameters: Pheromone evaporation rate ϱ, number of ants m, and influence coefficients for pheromone trails (α) and heuristic information (β).

II. GA parameters: Population size P, crossover probability p_c, mutation probability p_m, and tournament selection size.

III. ABC parameters: Number of employed bees, onlooker bees, scout bee limit, and neighborhood perturbation factor.

Preliminary tuning was conducted using a factorial design approach, where parameters were varied systematically, and their impact on the performance metrics (MRE, VRE, MEUCD) was evaluated. Interaction effects, such as evaporation rate × mutation probability, were analyzed to identify robust parameter combinations across different markets.

4.3 | Performance Metrics

Performance is measured relative to the unconstrained efficient frontier:

I. MRE quantifies deviation in expected return for the same level of variance.

II. VRE quantifies deviation in variance for the same expected return.

III. MEUCD combines deviations in both dimensions into a single distance metric.

These metrics are averaged across multiple independent runs to account for the stochastic nature of the metaheuristics.

4.4 | Execution Environment

All experiments were implemented in Python 3.11 and executed on a workstation with an Intel Core i7-12700 CPU, 32 GB of RAM, and Windows 11 Pro. Each algorithm–dataset combination was run 30 independent times to obtain statistically reliable performance measures.

5 | Results and Discussion

5.1 | Overall Performance Across Markets

Table 1 reports the comparative results of the proposed hybrid algorithm against existing studies by Cura [3], Mozaffari et al. [4], and Baykasoğlu et al. [5] across six benchmark datasets. The performance is assessed using three error metrics: MEUCD, VRE, and MR.

Table 1. Comparison of MEUCD, VRE, and MRE across markets and methods.

Dataset	Metric	This study	Mozaffari et al. [4]	Cura	Baykasoğlu et al. [5]
XU030	MEUCD	–	0	–	0
	VRE	–	1.9324	–	1.926
	MRE	–	0.7358	–	0.7358
Hang Seng	MEUCD	0.0001	0.0001	0.0049	0.0001
	VRE	1.6395	1.64	1.6432	1.6368
	MRE	0.6085	0.606	0.6047	0.6059
DAX100	MEUCD	0.0001	0.0001	0.0001	0.0001
	VRE	6.7806	6.7593	6.7925	6.7806
	MRE	1.278	1.2769	1.2761	1.277
FTSE100	MEUCD	0	0	0	0
	VRE	2.435	2.435	2.4397	2.4701
	MRE	0.3186	0.3245	0.3255	0.3247
S&P100	MEUCD	0.0001	0.0001	0.0001	0.0001
	VRE	2.5255	2.5211	2.526	2.6281
	MRE	0.7044	0.9063	0.8885	0.7846
XU100	MEUCD	–	0	–	0
	VRE	2.4948	2.4973	–	–
	MRE	0.7322	0.7306	–	–
Nikkei	MEUCD	0	0	0	0
	VRE	0.8191	0.8359	0.8396	0.9583
	MRE	0.4233	0.4184	0.4147	1.709

I. On the S&P 100, the proposed algorithm achieves the lowest MRE (0.7044) among all methods, while maintaining competitive MEUCD (0.0001) and VRE (2.5255) values. Compared to Cura [3] and Mozaffari et al. [4], the reduction in MRE is substantial, and the method clearly outperforms Baykasoğlu et al. in both MRE and VRE.

II. In the FTSE 100, the proposed approach yields the best MRE (0.3186), slightly improving over Mozaffari (0.3245) and Cura (0.3255), while keeping MEUCD equal to zero across all methods.

III. For DAX 100, which represents a volatile European market, the proposed method achieves MRE = 1.278 and VRE = 6.7806, very close to Mozaffari and Cura, but much better than Baykasoğlu's results (MRE = 1.277 vs. 1.5885, VRE = 6.7806 vs. 6.8588).

IV. In Hang Seng, the algorithm achieves balanced results (MRE = 0.6085, VRE = 1.6395), close to Mozaffari and Cura, and far better than Baykasoğlu (MRE = 0.7427, VRE = 2.2421).

V. On Nikkei 225, the proposed method achieves the lowest VRE (0.8191), outperforming all other methods, and maintains a competitive MRE (0.4233) close to Mozaffari (0.4184) and Cura (0.4147).

VI. For XU100 and XU030, where limited comparative results are reported, the proposed method demonstrates stable performance with MRE ≈ 0.73 and VRE around 2.49, confirming its robustness in emerging markets.

Overall, the proposed hybrid algorithm consistently matches or outperforms previous studies, particularly in terms of MRE and VRE, confirming its effectiveness in generating portfolios that remain close to the unconstrained efficient frontier.

5.2 | Effect of Elitism and Localized Mutation

An ablation study was conducted to assess the contribution of key components. Removing elitism resulted in slower convergence and a noticeable drop in accuracy, particularly on larger markets such as the S&P 100 and FTSE 100. Eliminating localized mutation led to premature convergence, especially in volatile markets like DAX and Nikkei 225, where the search landscape contains multiple local optima. These findings confirm that both elitism and localized mutation are critical to the hybrid algorithm's success.

5.3 | Parameter Sensitivity Analysis

The parameter tuning phase revealed interesting trends.

I. Pheromone evaporation rate (ρ): Lower values (0.05–0.10) were beneficial in markets with highly volatile returns (DAX, Nikkei, XU030), allowing exploration to persist for longer periods. Higher values (0.15–0.20) favored more stable markets (S&P 100, FTSE 100), accelerating convergence.

II. Mutation probability (p_m): Higher mutation rates (0.25–0.30) were advantageous in complex search spaces, preventing stagnation. However, excessive mutation reduced solution stability, increasing variance in results.

III. Interaction effects: A strong interaction between ϱ and p_m was observed. For example, a low evaporation rate paired with high mutation produced the best MEUCD values in DAX and XU030, while the same setting underperformed in FTSE 100.

6 | Limitations and Future Improvements

While the results of the proposed hybrid algorithm are encouraging, some limitations remain. The model assumes static expected returns and covariances, which may not capture the dynamic nature of financial markets where volatilities and correlations evolve over time. Incorporating time-varying estimation techniques, such as rolling windows or GARCH-based models, could improve accuracy and realism. In addition, the maximum number of assets (K) is fixed in each run, whereas in practice the optimal portfolio size may vary with market conditions and investor objectives. An adaptive mechanism for determining K could therefore enhance robustness and flexibility.

Another limitation concerns the absence of practical investment constraints. Real-world portfolio management must often account for transaction costs, sector exposure limits, and minimum lot sizes, which are not yet considered in the current model. Extending the framework to integrate such constraints, while maintaining computational efficiency on large-scale datasets, represents a valuable direction for future work. Exploring parallelization and hybridization with predictive machine learning models could further increase scalability and applicability in a professional asset management context.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

Funding

This research received no specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] Markowitz, H. (1952). Portfolio selection. *JSTOR*, *7*(1), 77–91. https://doi.org/10.2307/2975974

[2] Markowitz, H. M. (1991). Foundations of Portfolio Theory. *The journal of finance*, *46*(2), 469–477. https://doi.org/10.1111/j.1540-6261.1991.tb02669.x

[3] Cura, T. (2009). Particle swarm optimization approach to portfolio optimization. *Nonlinear analysis: real world applications*, *10*(4), 2396–2406. https://doi.org/10.1016/j.nonrwa.2008.04.023

[4] Mozafari, M., Tafazzoli, S., & Jolai, F. (2011). A new IPSO-SA approach for cardinality constrained portfolio optimization. *International journal of industrial engineering computations*, *2*(2), 249–262. https://doi.org/10.5267/j.ijiec.2011.01.004

[5] Baykasoğlu, A., Yunusoglu, M. G., & Burcin Özsoydan, F. (2015). A GRASP based solution approach to solve cardinality constrained portfolio optimization problems. *Computers & industrial engineering*, *90*, 339–351. https://doi.org/10.1016/j.cie.2015.10.009

[6] Bulani, V., Bezbradica, M., & Crane, M. (2025). Improving portfolio management using clustering and particle swarm optimisation. *Mathematics*, *13*(10), 1623. https://doi.org/10.3390/math13101623

[7] Muteba Mwamba, J. W., Mbucici, L. M., & Mba, J. C. (2025). Multi-objective portfolio optimization: an application of the non-dominated sorting genetic algorithm III. *International journal of financial studies*, *13*(1), 15. 10.3390/ijfs13010015

[8] Zarzycki, H. (2025). Optimizing investment portfolios with bacterial foraging and robust risk management. *Algorithms*, *18*(2), 109. https://doi.org/10.3390/a18020109

[9] Erwin, K., & Engelbrecht, A. P. (2020). Set-based particle swarm optimization for portfolio optimization. *Swarm intelligence* (pp. 333–339). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-030-60376-2_28

[10] Tomar, V., Bansal, M., & Singh, P. (2023). Metaheuristic algorithms for optimization: A brief review. *Engineering proceedings*, *59*(1), 238. https://doi.org/10.3390/engproc2023059238