Annals of Optimization With Applications

www.anowa.reapress.com

Ann. Optim. Appl. Vol. 1, No. 4 (2025) 301–308.

Paper Type: Original Article

A Framework for Efficiency Analysis and Resource Allocation Using Inverse DEA under Uncertainty

Shabnam Razavyan*

Department of Mathematics, South Tehran Branch, Islamic Azad University, Tehran, Iran; sh_razavyan@azad.ac.ir.

Citation:

Received: 5 April 2025 *Revised: 13 July 2025* *Accepted: 16 October 2025*	Razavyan, R. (2025). A framework for efficiency analysis and resource allocation using inverse dea under uncertainty. *Annals of Optimization With Applications*, *1*(4), 301-308.

Abstract

Inverse Data Envelopment Analysis (Inv-DEA) determines the input increase required for a Decision-Making Unit (DMU) to increase its outputs, or the additional outputs attainable when inputs rise to a predetermined level, all while maintaining current efficiency. Traditional Inv-DEA models assume precise (crisp) data. However, real-world applications often involve imprecise inputs and outputs due to inherent complexity and uncertainty. This study introduces a methodological framework that integrates uncertainty theory based on belief degrees into both input-oriented and output-oriented Inv-DEA models. The proposed models are applied to resource allocation problems. We employ uncertain programming techniques to convert the resulting nonlinear uncertain Inv-DEA models into solvable deterministic linear programs. This transformation is achieved through two key approaches: For models with uncertain variables in the objective function, we use the expected value criterion to optimize average performance. For models with uncertain constraints, we apply chance-constrained programming. This framework effectively addresses essential uncertainty, enabling computational solution via standard linear programming techniques. To validate the practical applicability and effectiveness of the proposed inverse DEA methodology for resource allocation, we conduct an efficiency evaluation using a numerical example.

Keywords: Data envelopment analysis, Inverse data envelopment analysis, Efficiency, Uncertainty, Resource allocation.

1 | Introduction

The objective of Inverse Data Envelopment Analysis (Inv-DEA) is to estimate the inputs and/or outputs of Decision-Making Units (DMUs) such that their current efficiency remains unchanged while the targeted input and/or output resources are modified [1]. In recent years, due to its wide range of applications in sectors such as business, banking, healthcare, and supply chain management, the Inv-DEA approach has gained increasing

Corresponding Author: sh_razavyan@azad.ac.ir
https://doi.org/10.48314/anowa.v1i4.63

popularity [2-5]. In this regard, Shiri Daryani et al. [4] proposed an Inv-DEA approach for merging DMUs with two-stage network structures. Their model estimated the inputs and intermediate products of two-stage DMUs to achieve various predetermined efficiency scores set by the decision-maker. A comprehensive review of recent theoretical and practical developments in Inv-DEA was subsequently provided by Emrouznejad et al. [2], who examined the evolution of theoretical models and their emerging applications across new domains. More recently, Ghiyasi et al. [5] developed a model that accommodates simultaneous perturbations of both inputs and outputs, thereby reflecting a more realistic scenario for many practical decision-making problems.

In real-world applications, the complexity of practical problems often introduces uncertainty into inputs and outputs. Applying deterministic inverse DEA models under such conditions may therefore yield unrealistic results. Consequently, data uncertainty has emerged as one of the pivotal challenges in efficiency assessment across most applied domains. This issue is particularly pronounced in cases where data cannot be measured precisely, such as the number of infected patients or death cases during a disease outbreak, which are typically estimated by experts. Moreover, as Liu [6] argues, probability theory and fuzzy set theory are not always suitable for handling this type of uncertain data. Therefore, incorporating uncertainty theory based on belief degrees introduced by Liu [7]into inverse DEA constitutes the main objective of this paper.

Uncertainty theory differs from probability theory, which requires large samples and precise distributions. When samples are unavailable, expert belief degrees are essential [8]. For fuzzy data, Liu and Liu [9] introduced credibility measures to address the non-self-dual nature of possibility measures in fuzzy DEA. Building on this, Gholami Golsefid et al. [10] combined inverse DEA with network structures using triangular fuzzy numbers. Huang and Chen [11] extended this by modeling simultaneous fuzzy and random uncertainties. The foundation of uncertain DEA was laid by Wen and Kang [12] and further developed by Wen [12]. Recently, Yang and Yang [13] proposed a chance-constrained network DEA framework for sustainable supplier selection under uncertainty. Despite advances in network Inv-DEA (Shiri Daryani et al., [4] and cost-revenue efficiency (Ghiyasi et al., [14]), no study has addressed inverse problems through Liu's uncertainty theory. This study fills that gap by employing uncertainty theory to model belief degrees for imprecise data, providing a robust axiomatic foundation for indeterminacy in inverse problems.

This study addresses the identified research gap by proposing a generalized methodology valid under the more realistic assumption of Variable Returns To Scale (VRS). In contrast to existing research that typically considers only a single orientation, our framework offers a comprehensive approach by encompassing both input-oriented and output-oriented uncertain Inv-DEA models. Moreover, we establish and prove theorems that apply uncertain programming techniques—namely expected value and chance-constrained programming to efficiently convert the proposed nonlinear uncertain models into deterministic linear programming problems that are readily solvable. It is worth emphasizing that uncertain inverse DEA estimates the inputs and/or outputs of a specific DMU while preserving its efficiency level and adjusting the targeted uncertain outputs and/or input resources. To the best of our knowledge, this paper presents the first unified methodology that integrates uncertain data into Inv-DEA under VRS for both input and output orientations. The main contributions of this study to the DEA literature can be summarized as follows:

I. the integration of uncertain data into both input-oriented and output-oriented inverse DEA models;

II. the introduction of a new optimality criterion, termed the weak Pareto solution, to derive necessary and sufficient conditions for estimating uncertain inputs and outputs; and

III. the development of theorems based on expected value and chance-constrained programming to transform the novel uncertain models into their deterministic equivalents.

The remainder of this paper is structured as follows. Section 2 presents the uncertain BCC model, which serves as the foundation for the proposed methodology. In Section 3, we develop inverse DEA models that incorporate uncertain data, accompanied by theorems based on expected value and chance-constrained programming to convert these novel uncertain models into their deterministic equivalents. Section 4 provides a numerical example to illustrate the practical applicability of the proposed approach. Finally, Section 5 concludes the paper and discusses directions for future research.

2|The Uncertain BCC Model

Consider n DMUs, where each DMU_j $(j = 1,\dots, n)$ consumes M uncertain inputs $(\tilde{x}_{ij}, i=1,\dots,m)$ to produce s uncertain outputs $(\tilde{y}_{rj}, r=1,\dots,s)$ For the DMU under evaluation, DMU_o, the input-oriented uncertain BCC model is formulated as follows:

$$\begin{aligned}
&\theta_o^* = \min\ \theta_o \\
&\text{s.t.}\quad M\left\{\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij} \le \theta_o\tilde{x}_{io}\right\} \ge \alpha,\quad i=1,\dots,m, \\
&\qquad M\left\{\sum_{j=1}^{n}\lambda_j\tilde{y}_{rj} \ge \tilde{y}_{ro}\right\} \ge \alpha,\quad r=1,\dots,s\ , \\
&\qquad \sum_{j=1}^{n}\lambda_j = 1, \\
&\qquad \lambda_j \ge 0, j=1,\dots,n.
\end{aligned} \tag{1}$$

To derive the deterministic equivalent of *Model (1)* based on belief degree, the uncertain variable $\xi = \sum_{j=1, j\ne o}^{n}\lambda_j\tilde{x}_{ij} - \tilde{x}_{io}(\theta_o - \lambda_o)$ is strictly increasing with respect to $\tilde{x}_{ij}, j \ne o, j=1,\dots,n$ and strictly decreasing with respect to $\tilde{x}_{io}$. All uncertain inputs and outputs in our framework are assumed to follow regular uncertainty distributions. Since these distributions are continuous and strictly increasing, their inverse functions $\varphi_{ij}^{-1}(\alpha)$ and $\psi_{rj}^{-1}(\alpha)$ exist and are uniquely defined for any belief degree his invertibility property enables T $\alpha \in (0,1)$. the transformation of chance constraints into their deterministic equivalents. Consequently, *Model (1)* can be expressed in its deterministic form as follows:

$$\begin{aligned}
&\theta_o^* = \min\ \theta_o \\
&\text{s.t.}\quad \sum_{j=1, j\ne o}^{n}\lambda_j\varphi_{ij}^{-1}(\alpha) + \lambda_o\varphi_{io}^{-1}(1-\alpha) \le \theta_o\varphi_{io}^{-1}(1-\alpha),\quad i=1,\dots,m \\
&\qquad \sum_{j=1, j\ne o}^{n}\lambda_j\psi_{rj}^{-1}(1-\alpha) + \lambda_o\psi_{ro}^{-1}(\alpha) \ge \psi_{ro}^{-1}(\alpha),\quad r=1,\dots,s \\
&\qquad \theta_o - \lambda_o \ge 0 \\
&\qquad \sum_{j=1}^{n}\lambda_j = 1 \\
&\qquad \lambda_j \ge 0, j=1,\dots,n.
\end{aligned} \tag{2}$$

For the DMUo as under evaluation DMU, the output-oriented uncertain BCC model is as follows:

$$\begin{aligned}
&\theta_o^* = \min\ \theta_o \\
&\text{s.t.}\ \sum_{j=1, j\neq o}^{n} \lambda_j \varphi_{ij}^{-1}(\alpha) + \lambda_o \varphi_{io}^{-1}(1-\alpha) \le \theta_o \varphi_{io}^{-1}(1-\alpha),\ \ i = 1,\dots,m \\
&\sum_{j=1, j\neq o}^{n} \lambda_j \psi_{rj}^{-1}(1-\alpha) + \lambda_o \psi_{ro}^{-1}(\alpha) \ge \psi_{ro}^{-1}(\alpha),\ \ r = 1,\dots,s \\
&\theta_o - \lambda_o \ge 0 \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0, j = 1,\dots,n.
\end{aligned} \tag{3}$$

Similarly, the deterministic form of *Model (3)* based on belief degree is as:

$$\begin{aligned}
&z_o^* = \max\ z_o \\
&\text{s.t.}\ \sum_{j=1, j\neq o}^{n} \lambda_j \varphi_{ij}^{-1}(\alpha) + \lambda_o \varphi_{io}^{-1}(1-\alpha) \le \varphi_{io}^{-1}(1-\alpha),\ \ i = 1,\dots,m \\
&\sum_{j=1, j\neq o}^{n} \lambda_j \psi_{rj}^{-1}(1-\alpha) + \lambda_o \psi_{ro}^{-1}(\alpha) \ge z_o \psi_{ro}^{-1}(\alpha),\ \ r = 1,\dots,s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0, j = 1,\dots,n.
\end{aligned} \tag{4}$$

3 | Inverse Models under Uncertainty

This section addresses the inverse DEA problem in the presence of uncertain data. Consider a set of n DMUs, where DMU_o is the DMU under evaluation. Assuming its uncertain inputs are increased to a predetermined level, we seek to determine the required increase in its uncertain outputs such that DMU_o maintains its current efficiency relative to the other DMUs. Suppose that DMU_o changes its inputs from $\tilde{x}_{io}(i = 1,\dots,m)$ to $\tilde{\alpha}_{io} = \tilde{x}_{io} + \Delta\tilde{x}_{io} = \tilde{x}_{io} + \eta_i \tilde{x}_{io}(i = 1,\dots,m)$, where $\eta_i \ge 0$. To estimate the new output level, i.e. $\tilde{\beta}_{ro} = \tilde{y}_{ro} + \Delta\tilde{y}_{ro} = \tilde{y}_{ro} + \delta_r \tilde{y}_{ro}(r = 1,\dots,s)$, where $\delta_r \ge 0$. Based on the belief degree, the following multi-objective programming problem is formulated:

$$\begin{aligned}
&z_o^* = \max\ z_o \\
&\text{s.t.}\ \sum_{j=1, j\neq o}^{n} \lambda_j \varphi_{ij}^{-1}(\alpha) + \lambda_o \varphi_{io}^{-1}(1-\alpha) \le \varphi_{io}^{-1}(1-\alpha),\ \ i = 1,\dots,m \\
&\sum_{j=1, j\neq o}^{n} \lambda_j \psi_{rj}^{-1}(1-\alpha) + \lambda_o \psi_{ro}^{-1}(\alpha) \ge z_o \psi_{ro}^{-1}(\alpha),\ \ r = 1,\dots,s \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0, j = 1,\dots,n.
\end{aligned} \tag{5}$$

let z_o^* represent the pre-perturbation efficiency score of DMUo obtained from *Model (3)*. To determine the new output level $\tilde{\beta}_{ro} = \tilde{y}_{ro} + \hat{\delta}_r \tilde{y}_{ro}(r = 1,\dots,s)$ for DMUo, it is necessary to transform *Model (5)* into its deterministic equivalent. This transformation is achieved by applying the following theorem.

Theorem 1. Let $\tilde{x}_{i1},\dots,\tilde{x}_{in}$, for $i=1,\dots,m$ be independent uncertain inputs with regular uncertainty distributions $\varphi_{i1}(\alpha),\dots,\ \varphi_{in}(\alpha)$, respectively, and $\tilde{y}_{r1},\dots,\tilde{y}_{rn}$, for $r=1,\dots,s$ be independent uncertain outputs with regular uncertainty distributions $\psi_{r1}(\alpha),\dots,\ \psi_{rn}(\alpha)$, respectively. Then, the uncertain *Model (5)* is equivalent to the following model:

$$\begin{aligned}
&\max\ E[\Delta\tilde{y}_o]=\sum_{r=1}^{s}\delta_r\int_0^1\psi_{ro}^{-1}(\alpha)d\alpha\\
&\text{s.t.}\ \sum_{j=1,j\neq o}^{n}\lambda_j\varphi_{ij}^{-1}(\alpha)+\lambda_o\varphi_{io}^{-1}(1-\alpha)\le(1+\eta_i)\varphi_{io}^{-1}(1-\alpha),\ \ i=1,\dots,m\\
&\quad\ \sum_{j=1,j\neq o}^{n}\lambda_j\psi_{rj}^{-1}(1-\alpha)+\lambda_o\psi_{ro}^{-1}(\alpha)\ge z_o^*(1+\delta_r)\psi_{ro}^{-1}(\alpha),\ \ r=1,\dots,s\\
&\quad\ \delta_r\psi_{ro}^{-1}(\alpha)\ge 0,\ \ r=1,\dots,s\\
&\quad\ \sum_{j=1}^{n}\lambda_j=1\\
&\quad\ \delta_r,\lambda_j\ge 0, r=1,\dots,s, j=1,\dots,n.
\end{aligned}\tag{6}$$

Proof: The function $\Psi_o=\sum_{r=1}^{s}\delta_r\tilde{y}_{ro}$ is strictly increasing with respect to $\delta_r\tilde{y}_{ro}$. Therefore, $\Psi_o^{-1}=\sum_{r=1}^{s}\delta_r\psi_{ro}^{-1}(\alpha)$ and $E[\Delta y_o]=\sum_{r=1}^{s}\delta_r\int_0^1\psi_{ro}^{-1}(\alpha)d\alpha$. On the other hand, $\tilde{\alpha}_{io}=\tilde{x}_{io}+\Delta\tilde{x}_{io}=\tilde{x}_{io}+\eta_i\tilde{x}_{io}$ for $i=1,\dots,m$ where $\eta_i\ge 0$ Since $1-\lambda_o+\eta_i\ge 0$ and $\Phi_i=\sum_{j=1,j\neq o}^{n}\lambda_j\tilde{x}_{ij}-(1-\lambda_o+\eta_i)\tilde{x}_{io}\le 0, i=1,\dots,m$ is strictly increasing by $\tilde{x}_{ij}$ and strictly decreasing by $\tilde{x}_{io}$ for $i=1,\dots,m$. Therefore, the constraints $M\left\{\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}\le\tilde{\alpha}_{io}\right\}\ge\alpha, i=1,\dots,m$ hold if and only if $\sum_{j=1,j\neq o}^{n}\lambda_j\varphi_{ij}^{-1}(\alpha)+\lambda_o\varphi_{io}^{-1}(1-\alpha)\le(1+\eta_i)\varphi_{io}^{-1}(1-\alpha), i=1,\dots,m$.

Definition 1. Let $(\delta,\lambda)=(\delta_1,\dots,\delta_s,\lambda_1,\dots,\lambda_n)$ be a feasible solution of *Model (6)*. It is said that (δ,λ) is a α-weak Pareto solution of *Model (6)* if there is no feasible solution $(\bar{\delta},\bar{\lambda})$ such that $\bar{\delta}_r>\delta_r, r=1,\dots,s$.

Theorem 2. The objective function of *Model (6)* is a decreasing function of α.

Proof: The proof is easy and we leave it (Wen, 2015).

The following theorem guarantees that the efficiency score of $(\tilde{x}_o,\tilde{y}_o)=(\tilde{x}_{1o},\dots,\tilde{x}_{mo},\tilde{y}_{1o},\dots,\tilde{y}_{so})$ and $(\tilde{\alpha}_o,\tilde{\beta}_o)=(\tilde{\alpha}_{1o},\dots,\tilde{\alpha}_{mo},\tilde{\beta}_{1o},\dots,\tilde{\beta}_{so})$ are the same.

Theorem 3. Suppose that z_o^* is the optimal solution of *Model (5)* and $(\hat{\delta},\hat{\lambda})$ is a α-weak Pareto solution of *Model (5)*. Then, the optimal value of the following model is z_o^*, too.

$$\begin{aligned}
&\max\ \bar{z}\\
&\text{s.t.}\ M\left\{\sum_{j=1}^{n}\lambda_j\tilde{x}_{ij}\le\tilde{\alpha}_{io}\right\}\ge\alpha,\ \ i=1,\dots,m\\
&\quad\ M\left\{\sum_{j=1}^{n}\lambda_j\tilde{y}_{rj}\ge\bar{z}(\tilde{y}_{ro}+\hat{\delta}_r\tilde{y}_{ro})\right\}\ge\alpha,\ \ r=1,\dots,s\\
&\quad\ \sum_{j=1}^{n}\lambda_j=1\\
&\quad\ \lambda_j\ge 0, j=1,\dots,n.
\end{aligned}\tag{7}$$

Proof: The above model evaluates $(\tilde{\alpha}_o, \tilde{\beta}_o)$, where $\tilde{\beta}_{ro} = \tilde{y}_{ro} + \hat{\delta}_r \tilde{y}_{ro}, \quad r = 1,\dots,s$. *Model (7)* has optimal solution with finite value. Therefore, let $(\lambda^*, \overline{z}_o^*)$ be its optimal solution. The α-weak Pareto solution $(\hat{\delta}, \hat{\lambda})$ of *Model (5)* holds the following constraints:

$$
\begin{aligned}
& M\left\{\sum_{j=1}^{n} \hat{\lambda}_j \tilde{x}_{ij} \le \tilde{\alpha}_{io}\right\} \ge \alpha, \quad i = 1,\dots,m \\
& M\left\{\sum_{j=1}^{n} \hat{\lambda}_j \tilde{y}_{rj} \ge z_o^* (\tilde{y}_{ro} + \hat{\delta}_r \tilde{y}_{ro})\right\} \ge \alpha, \quad r = 1,\dots,s \\
& M\left\{\tilde{y}_{ro} + \hat{\delta}_r \tilde{y}_{ro} \ge \tilde{y}_{ro}\right\} \ge \alpha, \quad r = 1,\dots,s \\
& \sum_{j=1}^{n} \hat{\lambda}_j = 1 \\
& \hat{\lambda}_j \ge 0, j = 1,\dots,n.
\end{aligned}
\tag{8}
$$

Therefore, $(\hat{\lambda}, z_o^*)$ is a feasible solution of *Model (7)* and $\overline{z}_o^* \ge z_o^*$. If $\overline{z}_o^* > z_o^*$, then, there is a $k > 1$ such that $\overline{z}_o^* = k z_o^*$ and $\overline{z}_o^* (\tilde{y}_{ro} + \hat{\delta}_r \tilde{y}_{ro}) > z_o^* (\tilde{y}_{ro} + \hat{\delta}_r \tilde{y}_{ro})$ for $r = 1,\dots,s$.

Hence, there is a feasible solution of *Model (7)*, say $(\overline{\delta}, \hat{\lambda})$, such that $\overline{\delta}_r = k\hat{\delta}_r > \hat{\delta}_r$. This is a contradiction. Because $(\hat{\delta}, \hat{\lambda})$ is a α-weak Pareto solution of *Model (7)*. Therefore, we have $\overline{z}_o^* = z_o^*$ and it completes the proof.

4 | Numerical Example

To illustrate the practical applicability of the proposed uncertain inverse DEA framework, we present a numerical example involving five DMUs. Each DMU consumes three uncertain inputs to produce three uncertain outputs. The input data are characterized by linear uncertainty distributions, denoted as $L(a,b)$, while the output data follow zigzag uncertainty distributions, denoted as $Z(a,b,c)$. The complete dataset is presented in *Table 1*.

Table 1. Input and output data for the five DMUs.

DMU	1	2	3	4	5
Input1	L(12,16)	L(19,22)	L(30,33)	L(20,23)	L(10,12)
Input2	L(15,18)	L(30,33)	L(40,47)	L(10,11)	L(10,12)
Input3	L(25,28)	L(10,11)	L(30,36)	L(10,12)	L(10,11)
Output1	Z(25,26,27)	Z(40,45,48)	Z(20,22,24)	Z(40,43,45)	Z(100,110,120)
Output2	Z(26,27,28)	Z(80,90,96)	Z(20,21,22)	Z(44,46,50)	Z(50,51,56)
Output3	Z(26,28,31)	Z(90,100,108)	Z(30,31,34)	Z(50,55,59)	Z(80,90,96)

First, the efficiency status of each DMU is determined using the uncertain BCC model presented in Section 2. As reported in *Table 2*, DMUs 2, 3, and 5 are identified as efficient, while DMUs 1 and 4 are classified as inefficient. To demonstrate the inverse DEA procedure under uncertainty, we consider a scenario wherein the average values of the uncertain inputs for each DMU are increased by 5%. The objective is to estimate the corresponding increase in the average values of the uncertain outputs such that each DMU maintains its current efficiency level. The results obtained from solving the proposed uncertain inverse models are summarized in *Table 2*.

Table 2. Efficiency status and new inputs and estimated new outputs.

DMU	Status	New Input1	New Input2	New Input3	New Output1	New Output2	New Output3
1	Inefficient	14.7	17.325	27.825	26.6252	27.6492	28.9293
2	Efficient	21.525	33.075	11.025	44.5	89	99.5
3	Efficient	33.075	45.675	34.65	22	21	31.5
4	Inefficient	22.575	11.025	11.55	47.8479	52.0451	61.2789
5	Efficient	11.025	11.55	11.025	110	52	89

As shown in *Table 2*, the estimated output values vary according to the efficiency status of each DMU. For efficient DMUs (2, 3, and 5), the increase in outputs is directly proportional to the input increase, reflecting their optimal performance. In contrast, for inefficient DMUs (1 and 4), the output adjustments account for both the input increase and the potential for efficiency improvement, resulting in output estimates that differ from a simple proportional rule.

This numerical example demonstrates the capability of the proposed uncertain inverse DEA framework to handle imprecise data characterized by uncertainty distributions while providing meaningful estimates for resource allocation problems. The results confirm that the methodology effectively preserves efficiency levels while accommodating data uncertainty through the application of uncertainty theory and deterministic transformation techniques.

5 | Conclusion

This study developed a novel methodological framework for inverse DEA under uncertainty, grounded in the concept of belief degrees for handling imprecise data. The primary theoretical contribution lies in introducing the weak Pareto solution concept, which establishes the necessary and sufficient conditions for estimating uncertain inputs and outputs. From a methodological perspective, we formulated uncertain inverse DEA models under the VRS assumption, thereby capturing a more realistic characterization of production processes in sectors such as healthcare. By applying the expected value operator to the objective function and employing chance-constrained programming to handle constraints with predefined confidence levels, we successfully transformed the proposed nonlinear uncertain models into solvable deterministic linear programs. The numerical example demonstrated the practical applicability of the proposed framework, illustrating how input increases translate into corresponding output adjustments while preserving the efficiency levels of DMUs. The results confirmed that the methodology effectively accommodates data uncertainty and provides meaningful estimates for resource allocation problems.

Several directions for future research emerge from this study. First, incorporating cost and revenue efficiency into the uncertain inverse DEA framework would extend its applicability to financial performance analysis. Second, applying the proposed methodology to analyze mergers and acquisitions under uncertainty represents a promising avenue, particularly in industries characterized by imprecise data. Third, expanding the indicator sets in empirical applications, such as healthcare, would enable more comprehensive performance assessments by capturing additional dimensions of organizational performance. Finally, exploring alternative uncertainty distributions and solution concepts could further enrich the theoretical foundations of uncertain inverse DEA.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

Funding

This research received no specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] Wei, Q., Zhang, J., & Zhang, X. (2000). An inverse DEA model for inputs/outputs estimate. *European journal of operational research*, *121*(1), 151–163. https://doi.org/10.1016/S0377-2217(99)00007-7

[2] Emrouznejad, A., Yang, G., & Amin, G. R. (2019). A novel inverse DEA model with application to allocate the CO2 emissions quota to different regions in Chinese manufacturing industries. *Journal of the operational research society*, *70*(7), 1079–1090. https://doi.org/10.1080/01605682.2018.1489344

[3] Liu, M., & CHAN, S. H. (2024). Optimizing supply chain efficiency using cross-efficiency analysis and inverse dea models. *WORLD*, *7*(5), 157–165. https://doi.org/10.53469/wjimt.2024.07(05).20

[4] Shiri Daryani, Z., Tohidi, G., Daneshian, B., Razavyan, S., & Hosseinzadeh Lotfi, F. (2021). Inverse DEA in two-stage systems based on allocative efficiency. *Journal of intelligent \& fuzzy systems*, *40*(1), 591–603. https://doi.org/10.3233/JIFS-200386

[5] Ghiyasi, M., Emrouznejad, A., & Amin, G. R. (2025). Generalizing inverse data envelopment analysis through directional distance function. *Expert systems with applications*, 129996. https://doi.org/10.1016/j.eswa.2025.129996

[6] Liu, B. (2009). Some research problems in uncertainty theory. *Journal of uncertain systems*, *3*(1), 3–10. https://www.researchgate.net/profile/Baoding-Liu/publication/228666965_Some_research_problems_in_uncertainy_theory/links/02e7e52836f150c344000000/Some-research-problems-in-uncertainy-theory.pdf

[7] Liu, B. (2007). Uncertainty theory. In *Uncertainty theory: a branch of mathematics for modeling human uncertainty* (pp. 1–79). Springer. https://doi.org/10.1007/978-3-642-13959-8_1

[8] Liu, B. (2004). Uncertainty theory: an introduction to its axiomatic foundation. *Physica-verlag, heidelberg*. https://doi.org/10.1007/978-3-540-39987-2

[9] Liu, B., & Liu, Y.-K. (2002). Expected value of fuzzy variable and fuzzy expected value models. *IEEE transactions on fuzzy systems*, *10*(4), 445–450. https://www.researchgate.net/profile/Baoding-Liu/publication/3336076_Liu_Y_Expected_Value_of_Fuzzy_Variable_and_Fuzzy_Expected_Value_Models_IEEE_Transactions_on_Fuzzy_Systems_10_445-450/links/00b7d520f2c082aca8000000/Liu-Y-Expected-Value-of-Fuzzy-Variable-and-Fuzzy-Expected-Value-Models-IEEE-Transactions-on-Fuzzy-Systems-10-445-450.pdf?_sg%5B0%5D=started_experiment_milestone&origin=journalDetail

[10] Gholami Golsefid, F., Jahani Sayyad Noveiri, M., & Kordrostami, S. (2024). Inverse network DEA incorporating cost and revenue efficiencies under fuzzy uncertainty. *Journal of mathematics*, *2024*(1), 8719643. https://doi.org/10.1155/2024/8719643

[11] Huang, L., & Chen, L. (2025). Fuzzy random multi-objective optimization using a novel mixed fuzzy random inverse DEA model in input-output production. *Journal of computational and applied mathematics*, *470*, 116717. https://doi.org/10.1016/j.cam.2025.116717

[12] Wen, M., Guo, L., Kang, R., & Yang, Y. (2014). Data envelopment analysis with uncertain inputs and outputs. *Journal of applied mathematics*, *2014*(1), 307108. https://doi.org/10.1155/2014/307108

[13] Yang, S., & Yang, X. (2025). A new uncertain two-stage network DEA model based on chance-constrained programming. *Journal of industrial and management optimization*, *21*(3), 2032–2051. https://doi.org/10.3934/jimo.2024161

[14] Ghiyasi, M., Nasrabadi, N., & Emrouznejad, A. (2025). Inverse DEA based on cost and revenue efficiency in the absence of price information. *Expert systems with applications*, *284*, 128028. https://doi.org/10.1016/j.eswa.2025.128028