Annals of Optimization With Applications

www.anowa.reapress.com

Ann. Optim. Appl. Vol. 1, No. 1 (2025) 50–65.

Paper Type: Original Article

Multi and Single-Period Uncertain Set Covering Location Problems with Robust Optimization Approach

Samaneh Rezvani[1,*], Foroogh Moeen Moghadas[2], Zohreh Dadi[2]

[1] Department of Science in Optimization, University of Bojnord, Iran; sam.rezvani513@gmail.com.

[2] Department of Applied Mathematics, University of Bojnord, Iran; f.moeen@ub.ac.ir; z.dadi@ub.ac.ir.

Citation:

Received: 17April 2024 *Revised: 21 July 2024* *Accepted: 28 September 2024*	Rezvani, S., Moeen Moghadas, F., & Dadi, Z. (2025). Multi-and single-period uncertain set covering location problems with robust optimization approach. *Annals of optimization with applications, 1*(1), 50-65.

Abstract

In recent years, covering location problems, owing to their numerous applications, have attracted significant attention. Since to the presence of uncertainty in real world data, in this paper a Single-Period Set Covering Location Problem (SPSCLP) as well as a Multi-Period Set Covering Location Problem (MPSCLP) under uncertainty in fixed establishment cost parameter are discussed by robust optimization approach. Various robust optimization approaches such as box approach, ellipsoidal approach and conservatism by adjustment approach are developed to this end. The robust counterparts of SPSCLP and MPSCLP are presented and then OR-Library dataset are used to analyze the models.

Keywords: Single-period set covering location problem, Multi-period set covering location problem, Uncertainty, Robust optimization.

1 | Introduction

The Set Covering Location Problem (SCLP) is one of the classical and important location problems first developed by Toregas et al. [1]. The aim of this problem is to find the minimum number (or least establishment cost) of facilities among potential facilities, so that each demand point is covered at least by one facility. In the classical set covering problem, the assignment of clients is specified by distance between facilities and clients. Karp [2] proved that the SCLP is NP-Hard. This problem can be applied to determine the location of emergency medical centers [3] or hospitals [4] and schedule the airline crew [5], [6], for instances. A review about this problem's applications can be found in [7], [8]. While the static problems deal only with a single-period, the dynamic ones concern planning over various time periods. In multi-period models, planning for a location to establish facilities or capacity of facilities in each period, are often

Corresponding Author: sam.rezvani513@gmail.comjfea

https://doi.org/10.48314/anowa.v1i1.54

considered, so that the time dimension is usually specified by a time-dependent variable. Multi-period location problem is essential when assignment costs, number of demands change during planning horizon. In this research, fixed establishment cost parameters in SCLP are assumed to change over various time periods.

Incorporating different aspects of uncertainty is also a main challenge in the real world situations. Some of the data and parameters of practical problems are not often precisely known. So, the uncertainty may not bring about the desirable optimum solution. Some of the common reasons for uncertainty are prediction error, measurement error and implementation error. In a facility location problem, for instance, the number of client demands, the radius of covering, inventory amount in stores, the 38 time and cost of travel may not be exactly specified. There are different approaches for optimization under uncertainty including stochastic approach [9], grey system [10], fuzzy optimization [3], probabilistic approach [11], Bayesian method [12], uncertainty theory [13] and robust optimization [14].

In this paper, the Multi-and Single-Period Set Covering Location Problems (SPSCLP, MPSCLP) under uncertainty in fixed establishment costs is discussed through the robust optimization approach. Here, we present the hard worst case and soft worst approaches. The introduced models are solved for OR-Library data by the CPLEX solver. In the rest of the paper, a brief literature review on multi- and single-period SCLPs is firstly presented in Section 2. Section 3 provides a mathematical model for SCLP and its robust counterpart models. A mathematical model for MPSCLP and its robust counterpart are introduced in Section 4. The comparison and analysis of the calculated results are presented in Section 5. Finally, the conclusions and suggestions for potential future researches are given in Section 6.

2 | Literature Review

The literature review on Uncertain Set Covering Location Problem (USCLP) is presented in Subsection 2.1. Subsection 2.2 summarizes researches concerning multi-period SCLP and finally a brief review on USCLP based on robust optimization approach is presented in Subsection 2.3.

2.1 | Set Covering Location Problem

Beraldi and Ruszczynski [15] considered a stochastic approach to study SCLP under uncertainty in constraint and proposed branch and bound algorithm to solve their model. Hwang [16] studied SCLP with the assumption of uncertainty in constraint by the fuzzy optimization approach. Chiang et al. [17] investigated the SCLP through the fuzzy approach. Saxena et al. [18] modelled the probabilistic SCLP under uncertainty in constraint and solved it by the cutting plane algorithm. Ahmed and Georgiou [7] studied the probabilistic SCLP under uncertainty caused by overcrowding; they applied a stochastic optimization approach and used some reliable inequalities to solve their model.

2.2 | Multi- and Single-Period Set Covering Location Problems

When the problem parameters vary over time, it is better to plan for more than one period. This results in a multi-period or dynamic location problem with parameters changing over time periods. A multi-period location problem is necessary as the problem parameters change over the planning horizon. Gunawardane [19] proposed a multi-period model for an SCLP to minimize the uncovered demands. He also took into account the possibilities that an open facility gets closed a closed one gets open before the end of the time period.

2.3 | Uncertain Set Covering Location Problem with Robust Optimization Approach

Several approaches have been proposed to deal with uncertainty. One of them is the robust optimization approach where for each problem containing uncertain parameters, a robust model is proposed; this is called the robust counterpart which would be solved instead of the main problem [20]. The robust optimization includes the hard worst case approach [21], [14], soft worst one [22], [23] and the realistic approach [24], [25] whose details can be found in Refs. [22], [26]. Pereira and Averbakh [27] discussed a probabilistic SCLP under uncertainty in the establishment costs of facilities with a scenario-based robust optimization approach. They used Benders decomposition method and a combination of Genetic and branch and bound algorithms to solve their proposed model. Lutter et al. [28] studied an uncertain probabilistic SCLP for emergency services and solved it based on a combination of robust optimization and probabilistic optimization approaches using the cutting plane algorithm.

3 | Robust Counterpart Model for Uncertain Set Covering Location Problem

To introduce the mathematical model of the USCLP and its counterpart, consider that the set of demand points and the set of potential location candidates for establishing facilities are denoted by I and J, respectively. Moreover, let f_j be the establishment cost of facilities in location $j \in J$, a_{ij} be the assignment parameter which is equal to one if the client $i \in I$ can be covered by a facility in location $j \in J$, and zero otherwise. The x_j is considered as the decision variable of the problem which is equal to one if a candidate location can provide service, and zero otherwise. The mathematical model is as follows [1].

$$\text{Min} \quad \sum_{j \in J} f_j x_j. \tag{1}$$

s. t.

$$\sum_{j \in J} a_{ij} x_j \geq 1, \quad i \in I. \tag{2}$$

$$x_j \in \{0,1\}, \qquad j \in J. \tag{3}$$

The objective function in *Eq. (1)* minimizes the establishment cost of facilities. The *Constraint (2)* guarantees that each client is covered by at least one facility. The definition domain of decision variable x_j is illustrated in *Constraint (3)*. Problem (1) is called, nominal set covering problem and it is NP-Hard [2]. Here, the fixed establishment costs $\tilde{f}_j$'s are assumed to be uncertain. So, the equivalent form of Problem (1) under uncertainty in the fixed establishment cost parameter can be rewriten as Problem (2):

$$\text{Min} \quad \tau. \tag{4}$$

s. t. (2), (3)

$$\sum_{j \in J} \tilde{f}_j x_j \leq \tau. \tag{5}$$

$$\tau \geq 0.$$

The Problem (2) is called the USCLP. The $\tilde{f}_j$ is an uncertain parameter taking random values in the interval of $[f_j - \overline{f_j}, f_j + \overline{f_j}]$ where $\overline{f_j}$ and f_j are the maximum non-negative perturbation and the nominal value of the uncertain parameter, respectively. Therefore, $\tilde{f}_j$ is defined as follows.

$$\tilde{f}_j = f_j + \xi_j \overline{f_j}. \tag{6}$$

Here, ξ_j being a stochastic parameter is the perturbation factor which varies in the interval of $[-1,1]$. For the facilities with the fixed establishment cost, $\overline{f_j}$ is assumed to be zero.

3.1 | Robust Counterpart Model of Set Covering Location Problem Based on Soyster Approach

The first robust optimization model was proposed in 1973 by Soyster [22]. Despite the perturbation in the uncertain parameters in the interval of $[-1,1]$, this model guarantees the robustness of the solutions. The solutions obtained by this model are conservative, i.e their robustness is guaranteed but the optimization of the main problem is lost greatly [22], [21]. Concerning the uncertain fixed establishment cost parameters, if $|\xi_j|$ takes values between 0 to ε $(0 < \varepsilon \leq 1)$, the sharing space of n perturbations forms an n-dimensional cube called the uncertainty box space due to its geometrical shape. For the Problem (2), the uncertainty box space is described as:

$$U^B = \{j \in J: \tilde{f}_j = f_j + \xi_j \bar{f}_j, |\xi_j| \leq \varepsilon\}. \quad (7)$$

The uncertain set *Eq. (7)* enforces the stochastic variables ξ_j to be less than ε. For each $\tilde{f}_j \in [f_j - \bar{f}_j, f_j + \bar{f}_j]$, we have

$$\sum_{j \in J} f_j x_j \leq \max_{f_j \in [f_j - \bar{f}_j, f_j + \bar{f}_j]} \left\{ \sum_j f_j x_j \right\} = \max_{\xi_j \in [-1,1]} \left\{ \sum_j \left(f_j + \xi_j \bar{f}_j\right) x_j \right\}. \quad (8)$$

According to U^B structure and $|\xi_j| \leq \varepsilon$ we have

$$\max_{\xi_j \in [-1,1]} \left\{ \sum_j \left(f_j + \xi_j \bar{f}_j\right) x_j \right\} \leq \sum_j f_j x_j + \varepsilon \sum_{j \in U^B} \left|\bar{f}_j x_j\right| = \sum_j f_j x_j + \varepsilon \sum_{j \in U^B} \bar{f}_j \left|x_j\right| = \sum_j f_j x_j + \varepsilon \sum_{j \in U^B} \bar{f}_j x_j, \quad (9)$$

where the last equality has been written considering the non-negative nature of $\bar{f}_j$ parameter and x_j variable. So, the robust counterpart model based on the uncertainty set *Eq. (7)* for a USCLP would be in the form of Problem (3):

$$\begin{aligned} &\text{Min} \quad \tau. \\ &\text{s.t. } (2), (3), (5) \\ &\sum_{j \in J} f_j x_j + \varepsilon \sum_{j \in U^B} \bar{f}_j x_j \leq . \end{aligned} \quad (10)$$

The *Constraint (4)* will be also satisfied by satisfaction of *Constraint (10)*. *Constraint (4)* will be satisfied by satisfying *Constraint (10)*.

Proposition 1. The optimum solution of the robust counterpart *Model (3)* is a feasible solution for the equivalent Problem (2).

Proof: assume that x^* is the optimum solution of the robust counterpart optimized model Problem (3). So according to *Relation (10)*, we have

$$\sum_j f_j x_j^* + \varepsilon \sum_{j \in U^B} \bar{f}_j \left|x_j^*\right| \leq \tau. \quad (11)$$

Based on $\xi_j \in [-1,1]$ and $-1 \leq \xi_j \leq 1$, one concludes that $\tilde{f}_j \geq f_j - \varepsilon \bar{f}_j$ and $\tilde{f}_j \leq f_j + \varepsilon \bar{f}_j$, i.e. the uncertain parameter of the problem, $\tilde{f}_j$, varies in the interval of $[f_j - \bar{f}_j, f_j + \bar{f}_j]$. The x^* is a feasible answer and robust in the uncertain interval, since:

$$\sum_j \tilde{f}_j x_j^* \leq \sum_j f_j x_j^* + \varepsilon \sum_{j \in U^B} \bar{f}_j x_j^* \leq \sum_j f_j x_j^* + \varepsilon \sum_{j \in U^B} \bar{f}_j \left|x_j^*\right| \leq \tau. \quad (12)$$

The last inequality is written according to the feasible and non-negative nature of x^*.

3.2 | Robust Counterpart Model of Uncertain Set Covering Location Problem Based on Ben-Tall and Nemirovski Approach

Criticizing the very conservative approach of Soyster, Ben-Tall and Nemirovski [29] stated that in an uncertain optimization problem, the possibility that all the uncertain parameters have their worst value simultaneously is very low. El-Ghaoui and Lebert [30] as well as Ben-Tall and Nemirovski [29] tried to improve the objective function value by reducing the uncertainty set and through the partial reduction of robustness. In the ellipsoidal approach, the uncertainty space for the fixed establishment cost parameters is introduced as follows.

$$U^E = \left\{ j \in J : \tilde{f}_j = f_j + \xi_j \bar{f}_j, \sum_{j\in J} \xi_j^2 \le \Omega^2 \right\}. \quad (13)$$

The set U^E is an ellipse; according to its structure, for each $\tilde{f}_j \in [f_j - \bar{f}_j, f_j + \bar{f}_j]$, we have

$$\sum_j \tilde{f}_j x_j = \sum_{j\in U^E} (f_j + \xi_j \bar{f}_j) x_j \le \sum_j f_j x_j + \varepsilon \sum_{j\in U^E} \bar{f}_j |x_j|. \quad (14)$$

Two non-negative variables y_j and z_j are defined as $0 \le x_j \le z_j + y_j$ and $\bar{f}$ and z are the vectors including $\bar{f}_j$ and z_j, respectively, i.e. $\bar{f} = (\bar{f}_1, \bar{f}_2, \dots, \bar{f}_n)$ and $z = (z_1, z_2, \dots z_n)$. Since $x_j \le z_j + y_j$:

$$\bar{f}_j x_j \le \bar{f}_j z_j + \bar{f}_j y_j \Rightarrow \sum_{j\in U^E} \bar{f}_j x_j \le \sum_{j\in U^E} \bar{f}_j z_j + \sum_{j\in U^E} \bar{f}_j y_j. \quad (15)$$

Also, we know that

$$\sum_{j\in U^E} \bar{f}_j x_j = \sqrt{\left(\sum_{j\in U^E} \bar{f}_j x_j \right)^2} \ge \sqrt{\sum_{j\in U^E} \bar{f}_j^2 x_j^2}. \quad (16)$$

Considering the non-negative parameter Ω, *Relation (15)* gives:

$$\sum_{j\in U^E} \bar{f}_j x_j \le \Omega \sqrt{\sum_{j\in U^E} \bar{f}_j^2 z_i^2} + \sum_{j\in U^E} \bar{f}_j y_i. \quad (17)$$

To obtain a feasible solution for the worst situation, i.e. *Relation (17)*, regarding the non-negative behavior of x_j variable, by defining two new variables, we have:

$$\sum_j f_j x_j + \varepsilon \sum_{j\in U^E} \bar{f}_j |x_j| \le \sum_j f_j x_j + \varepsilon \left[\sum_j \bar{f}_j y_j + \Omega \sqrt{\sum_{j\in U^E} \bar{f}_j^2 z_j^2} \right] \quad (18)$$

Moreover, if $x_j \ge 0$, *Relation (18)* is also established. The robust counterpart model for USCLP based on the uncertainty Set *Eq. (13)* is in the form of Problem (4):

$$\text{Min} \quad \tau.$$
$$\text{s.t. } \textit{Eqs. (2), (3), (5)} \quad (19)$$

$$\sum_{j\in J} f_j x_j + \varepsilon \left[\sum_{j\in U^E} \bar{f}_j y_j + \Omega \sqrt{\sum_{j\in U^E} \bar{f}_j^2 z_j^2} \right] \le \tau \quad (20)$$

$$x_j - z_j \le y_j.$$

$$z_j, y_j \geq 0. \tag{21}$$

The model Problem (4) is nonlinear. The advantage of the ellipsoid model over the box one is its less conservatism, however the final model is nonlinear and can not be solved through the methods applied to the discrete optimization problems [22].

Proposition 2. The solution of the robust counterpart Problem (4) is a feasible solution for the equivalent problem P_2.

Proof: it followed from *Relations (14)* and *(18)*.

The adjustable conservatism approach for a USCLP under uncertainty in the fixed facility establishment cost parameters is discussed in the following

3.3 | Robust Counterpart Model for Uncertain Set Covering Location Problem based on Bertsimas and Sim Approach

Bertsimas and Sim [22] introduced another uncertainty set with adjustable robustness. In this approach, Γ parameter, being not necessarily an integer, is used to adjust the robustness of the answers against their conserving level. The robust optimization approach guarantees the worst state. To prevent the violation of uncertain constraints, the uncertain parameters can vary in some intervals. In this method, it is assumed that at most $[\Gamma]$ number of parameters have uncertainty according to *Relation (6)* and the maximum perturbation of only another parameter such as t-th one, $\overline{f_t}$, has an uncertainty in the form of $(\Gamma - [\Gamma])\,\bar{f}_t$. The uncertainty set based on the adjustable conservatism approach for an SCLP is defined as:

$$U^{\Gamma} = \left\{ j \in J : \widetilde{f_j} = f_j + \xi_j \overline{f_j} : \sum_{j \in J} \lceil |\xi_j| \rceil \leq \Gamma, |\xi_j| \leq \varepsilon \right\}. \tag{22}$$

The above set is of all indices having uncertainty and $\lceil |\xi_j| \rceil$ is the smallest integer number equal to or greater than $|\xi_j|$. To guarantee a feasible solution and to prevent the violation of j-th constraint, the sum of all the uncertain parameters of that constraint should have perturbation at most equal to Γ parameter, i.e. $\sum_{j \in J} \lceil |\xi_j| \rceil \leq \Gamma$.*$\Gamma$* introduces a ratio of the number of uncertain parameters in *Constraint 6*. According to U^{Γ} structure, and for each $\tilde{f}_j \in [f_j - \overline{f_j}, f_j + \overline{f_j}]$ and maximum perturbation of t-th parameter, $(\Gamma - [\Gamma])\,\bar{f}_t$, we have:

$$\begin{aligned} \sum_j \widetilde{f_j}\, x_j &= \sum_{j \in U^{\Gamma}} \left(f_j + \xi_j \overline{f_j}\right) x_j \leq \sum_j f_j x_j + \varepsilon \sum_{j \in U^{\Gamma}} \overline{f_j}\, |x_j| \\ &\leq \sum_j f_j x_j + \varepsilon \max_{\{(S,t) | S \subseteq U^{\Gamma}, |S| = [\Gamma], t \in U^{\Gamma} \setminus S\}} \left\{ \sum_{j \in S} \overline{f_j} x_j + (\Gamma - [\Gamma]) \overline{f_t} x_t \right\}. \end{aligned} \tag{23}$$

The last equality in *Relation (23)* is written based on the non-negative nature of $\overline{f_t}$ and $\overline{f_j}$ parameters as well as the binary nature of x_j's. The robust counterpart model based on the uncertainty set for the SCLP is in the form of Problem (5):

$$\begin{aligned} &\text{Min } \tau \quad . \\ &\text{s.t. } (2), (3), (5) \\ &\sum_{j=1}^{n} f_j\, x_j + \varepsilon \max_{\{(S,t) | S \subseteq U^{\Gamma}, |S| = [\Gamma], t \in U^{\Gamma} \setminus S\}} \left\{ \sum_{j \in S} \overline{f_j} x_j + (\Gamma - [\Gamma]) \overline{f_t} x_t \right\} \leq \tau. \end{aligned} \tag{24}$$

Problem (5) is nonlinear. It transforms to a nominal problem, if $\Gamma = 0$ in *Constraint 24*, and it converts to Problem (1) (Soyster model), if $\Gamma = |U^{\Gamma}|$. Furthermore, for an integer Γ, the expression $(\Gamma - [\Gamma])$ in *Constraint*

24 vanishes and $\max_{\{(S,t)|S\subseteq U^{\Gamma},|S|=[\Gamma],t\in U^{\Gamma}\setminus S\}}\{\sum_{j\in S}\overline{f}_j x_j^*\}$ is used to protect j-th constraint and to guarantee a feasible robust answer.

To reformulate the problem in the form of a linear optimization one, consider the following proposition.

Proposition 3. Assume that the non-negative variable x^* is the optimum solution of *Eq. (25)*; then x^* would be the feasible solution of the Problem (6), as well *Eqs. (25)-(28).*

$$\beta\ (x^*,\Gamma) = \max_{\{(S,t)|S\subseteq U^{\Gamma},|S|=[\Gamma],t\in U^{\Gamma}\setminus S\}}\left\{\sum_{j\in S}\overline{f}_j|x_j^*| + (\Gamma-[\Gamma])\overline{f}_t x_j^*\right\}. \quad (25)$$

$$\beta\ (x^*,\Gamma) = \text{Max}\sum_{j\in U^{\Gamma}}\overline{f}_j x_j^* z_j. \quad (26)$$

$$\text{s.t}\ \sum_{j\in U^{\Gamma}} z_j \le \Gamma. \quad (27)$$

$$0 \le z_j \le 1. \quad (28)$$

Proof: the optimum solution of Problem (6) includes $[\Gamma]$ number of z_i's equal to one and a variable equal to $\Gamma-[\Gamma]$. Hence, the optimum solution of the Problem (6) is selected from the set of $\{(S,t)|S\subseteq U^{\Gamma},|S| = [\Gamma], t\in U^{\Gamma}\setminus S\}$ so that the objective function of *Eq. (25)* is maximized. Therefore, the two Problem (6) and (i) have the same optimum solutions.

In order to find the linear equivalent form of Problem (5), we discuss the duality of Problem (6). For this purpose, consider the dual variables corresponding to *Constraints (27)* and *(28)* as y and p_j, respectively. So the duality of Problem (6) may be written as:

$$\text{Min}\ \sum_{j\in U^{\Gamma}} p_j + \Gamma y. \quad (29)$$

$$\text{s.t}\ \ y + p_j \ge \overline{f}_j x_j^*. \quad j\in U^{\Gamma}. \quad (30)$$

$$p_j \ge 0. \quad j\in U^{\Gamma}. \quad (31)$$

$$y \ge 0. \quad (32)$$

According to *Proposition 3*, Problem (6) is feasible, so regarding the strong duality, Problem (7) is also feasible. Furthermore, based on *Proposition 3*, $\beta\ (x^*,\Gamma)$ value is equal to the value of the objective function of Problem (7). The Problem (8) is resulted by replacing the objective function of dual Problem (7) with $\max_{\{(S,t)|S\subseteq U^{\Gamma},|S|=[\Gamma],t\in U^{\Gamma}\setminus S\}}\{\sum_{j\in S}\overline{f}_j x_j + (\Gamma-[\Gamma])\overline{f}_t x_j\}$ in *Constraint (26)* and adding *Constraints (30)-(32)* to Problem (6). Problem (8) is called the robust counterpart of USCLP under uncertainty in the fixed establishment costs.

$$\text{Min}\ \ \tau. \quad \text{s.t.} \quad (33)$$

$$\sum_j f_j x_j + \varepsilon\left(\Gamma y + \sum_{j\in U^{\Gamma}} p_j\right) \le \tau. \quad (34)$$

$$y + p_j \ge \overline{f}_j. \quad j\in U^{\Gamma}. \quad (35)$$

$$y \ge 0. \quad p_j \ge 0. \quad (36)$$

$j \in U^{\Gamma}$.

In the rest of the paper, the robust counterpart models of MPSCLP under uncertainty in the fixed establishment cost parameters are discussed.

4 | Robust Counterpart Models for Uncertain Multi-Period Set Covering Location Problem

In multi-period or dynamic models, it is assumed that one or some of the parameters vary over various planning times. Due to budget limitations in the real world problems, there should be some plans applicable to various periods. The model parameters are defined as in Section 3. T denotes the time periods, f_{jt} parameter is the establishment cost of facility in location j in t-th period and a_{ijt} assignment parameter equals one, if client $i \in I$ can be covered by a facility located in $j \in J$ in period $(t = 1,\dots,T)$, and zero otherwise. Furthermore, x_{jt} is the decision variable of the problem being one if a facility is located at a candidate center $j \in J$ in period $(t = 1,\dots,T)$, and zero otherwise. The mathematical model of MPSCLP is defined as follows:

$$\text{Min} \quad \sum_{t=1}^{T}\sum_{j\in J} f_{jt}\, x_{jt}. \tag{37}$$

$$\text{s.t} \quad \sum_{j\in J} a_{ijt}\, x_{jt} \geq 1, \quad i \in I, t = 1,\dots,T. \tag{38}$$

$$x_{jt} \in \{0,1\} j \in J, t = 1,\dots,T. \tag{39}$$

The objective function and constraints are defined as in Section 3, with consideration of the time period parameter. Here, it is also assumed that the fixed establishment costs in t'th time period,representede by $\widetilde{f_{jt}}$, have uncertainty. So, to prevent appearance of an uncertain paremeter in the objective function of Problem (9), it is transferred to a distinct constraint. For this purpose, the objective function expression is placed in a distinct constraint equal and smaller than a variable τt, for example. Since the new objective function is the minimized sum of τt values and $\sum_{j\in J}\widetilde{f_{jt}}x_{jt}$ is equal and smaller than τ_t, the minimization of τ_t's sum results in Problem (9). Definition of objective function and constraints are similar to Section 3, considering time period parameters. The equivalent Problem (9), similar to USCLP, is formulated as follows:

$$\begin{aligned} &\text{Min} \quad \sum_{t=1}^{T} \tau_t. \\ &\text{s.t. Eqs. (38), (39)} \\ &\sum_{j\in J} \widetilde{f_{jt}} x_{jt} \leq \tau_t \; t = 1,\dots,T. \end{aligned} \tag{40}$$

$$\begin{aligned} &\tau_t \geq 0. \\ &t = 1,\dots,T. \end{aligned} \tag{41}$$

The Problem (10) is called the uncertain MPSCLP and it is assumed that $\widetilde{f_{jt}}$ is an uncertain parameter taking randomly values from the interval of $[f_{jt} - \overline{f_{jt}}, f_{jt} + \overline{f_{jt}}]$. According to Subsection 3.1, the uncertain box space for Problem (10) is defined as:

$$U_t^B = \{j \in J: \widetilde{f_{jt}} = f_{jt} + \xi_{jt}\overline{f_{jt}}, |\xi_{jt}| \leq \varepsilon\} t = 1,\dots,T. \tag{42}$$

Analogous to Subsection 3.1, the robust counterpart of MPSCLP under uncertainty in the fixed establishment costs is as follows:

$$\begin{aligned} &\text{Min} \quad \sum_{t=1}^{T} \tau_t. \\ &\text{s.t. Eqs. (38), (39), (41)} \\ &\sum_{j\in J} f_{jt}x_{jt} + \varepsilon \sum_{j\in U_t^B} \overline{f_{jt}}x_{jt} \le \tau_t t = 1,\dots,T. \end{aligned} \quad \textbf{(43)}$$

The single-period model analogous to *Propositions 1-3* is confirmed for multi-period models. According to Subsection 3.2, the ellipsoidal uncertainty space under uncertainty in the fixed facility establishment cost parameters over t'th period is introduced as:

$$U_t^E = \left\{ j \in J: \widetilde{f_{jt}} = f_{jt} + \xi_{jt}\overline{f_{jt}}, \sum_{j\in J} \xi_{jt}^2 \le \Omega^2 \right\} t = 1,\dots,T. \quad \textbf{(44)}$$

As in Subsection 3.2, by defining two non-negative variables y_{it} and z_{it}, the robust counterpart model for uncertain MPSCLP based on uncertainty set *Eq. (44)* is in the form of Problem (12):

$$\begin{aligned} &\text{Min} \quad \sum_{t=1}^{T} \tau_t. \\ &\text{s.t. Eq. (38), (39), (41)} \\ &\sum_{j\in J} f_{jt}x_{jt} + \varepsilon\left[\sum_{j\in U_t^E} \overline{f_{jt}}y_{jt} + \Omega\sqrt{\sum_{j\in U_t^E} \overline{f}_{jt}^2 z_{jt}^2} \right] \le \tau_t \qquad t = 1,\dots,T \quad \textbf{(45)} \\ &x_{jt} - z_{jt} \le y_{jt.} \\ &j \in J, t = 1,\dots,T. \quad \textbf{(46)} \\ &z_{jt}, y_{jt} \ge 0. \\ &j \in J, t = 1,\dots,T. \quad \textbf{(47)} \end{aligned}$$

Similar to Subsection 3.3, uncertainty set for uncertain MPSCLP can be defined as:

$$U_t^\Gamma = \left\{ j \in J: \widetilde{f_{jt}} = f_{jt} + \xi_{jt}\overline{f_{jt}}, \sum_{j\in J} \left[\left|\xi_{jt}\right|\right] \le \Gamma_t, \left|\xi_{jt}\right| \le \varepsilon \right\} t = 1,\dots,T. \quad \textbf{(48)}$$

According to Subsection 3.3, the linear robust counterpart model for uncertain MPSCLP based on conservatism by adjustment approach would be in the form of Problem (13):

$$\begin{aligned} &\text{Min} \quad \sum_{t=1}^{T} \tau_{t.} \\ &\text{s.t. Eqs. (38), (39), (41)} \\ &\sum_{j\in J} f_{jt}x_{jt} + \varepsilon\left(\Gamma_t y_t + \sum_{j\in U_t^\Gamma} p_{jt} \right) \le \tau_t t = 1,\dots,T. \quad \textbf{(49)} \\ &y_t + p_{jt} \ge \overline{f_{jt}}y_{jt}. \\ &j \in J, t = 1,\dots,T. \quad \textbf{(50)} \\ &y_t \ge 0. \\ &t = 1,\dots,T. \quad \textbf{(51)} \\ &p_{jt} \ge 0. \\ &j \in J, t = 1,\dots,T. \quad \textbf{(52)} \end{aligned}$$

5 | Comparison and Analysis of Calculated Results

Here, the performances of robust counterpart models of SPSCLP and MPSCLP are compared in two separate sections. For a better comparison, the data in OR-Library are used with 507 and 582 demand points and 50 and 100 demand points random data. The results are presented with four decimal digits. The computational experiments were carried out on an Intel (R) core (TM) i5 − 4300M processor with 8GB of RAM using version 25.0.2 of GAMS software and CPLEX one.

5.1 | Comparison and Analysis of Calculated Results of Uncertain Set Covering Location Problem

In this section, the performances of the robust counterpart models and the nominal models of SCLP are compared. For this purpose, the nominal *Model (1)* is compared with its three robust counterpart *Models (3), (4)* and *(8)*. To calculate $\tilde{f}_j$, ε is assumed to be 0.5 and ξ_j parameter is randomly constructed. For a better analysis, the robust counterpart models with $\overline{f_j} = 0{,}1 * f_j$ and $\overline{f_j} = 0{,}01 * f_j$ are evaluated. The results of numerical samples with the given parameters for random samples and OR-Library data are presented in ten columns of *Tables 1* and *2*, respectively.

The first left column represents the calculation method of $\overline{f_j}$ parameter with respect to the nominal value and the second column shows the number of demand points. The optimum value of the model's objective function (OPT), time in second (CPU), relative error compared to the nominal error (ERROR) and the number of facilities to cover the demand points in the solution of each model for the intended sample are given in the third column.

The fourth and fifth columns present the objective function values obtained by solving the nominal *Model (1)* and the robust counterpart model P_3 with the box approach, respectively. The sixth column shows the function values are obtained with the ellipsoidal approach Problem (4). The seventh to tenth columns list the objective function values obtained based on the adjustable conservatism approach Problem (8) with the conserving level of $\Gamma = 0.25, 0.5, 0.75, 1$, respectively.

Γ defines the relative number of uncertain parameters in *Relation (6)*. Since GAMS software is not capable of ellipsoidal models calculation for the samples of OR-Library, to discuss the accuracy and performance of the model, the results are considered for random samples, first. The analysis of the results obtained by solving random samples and OR-Library data are discussed in the following.

Table 1. Computational results of model P_1 and robust counterpart models P_3, P_4 and P_8 for random samples.

$\overline{f_j}$	Demand	Information for Solving the Model	Model Type	P_1	P_3	P_4	P_8			
							$\Gamma = 0,25$	$\Gamma = 0,5$	$\Gamma = 0,75$	$\Gamma = 1$
$0{,}1 * f_j$	50	OPT		133,0000	141,2500	134,1970	134,2500	135,5000	136,7500	138,0000
		CPU		0,0000	0,0160	0,0000	0,0000	0,1500	0,0000	0,0150
		ERROR		0,0000	0,0620	0,0090	0,0093	0,0188	0,0282	0,0376
		number of facilities to cover demand points		4	3	4	4	4	4	4
	100	OPT		34,0000	34,5840	34,0000	34,8500	35,7000	36,5500	37,4000
		CPU		0,0310	0000	0,0160	0,0320	0,0000	0,0160	0,0160
		ERROR		0,0000	0,0172	0,0000	0,0250	0,0500	0,0750	0,1000
		number of facilities to cover demand points		1	1	1	1	1	1	1
$0{,}01 * f_j$	50	OPT		133,0000	140,1250	133,1200	133,1250	133,2500	133,3750	133,5000
		CPU		0,0000	0,0000	0,0160	0,0000	0,0160	0,0160	0,0150
		ERROR		0,0000	0,0536	0,0009	0,0009	0,0019	0,0028	0,0038
		number of facilities to cover demand points		4	3	4	4	4	4	4
	100	OPT		34,0000	34,0580	34,0000	34,0850	34,1700	34,2550	34,3400
		CPU		0,0310	0,0160	0,0160	0,0160	0,0000	0,0160	0,0160
		ERROR		0,0000	0,0017	0,0000	0,0025	0,0050	0,0075	0,0100
		number of facilities to cover demand points		1	1	1	1	1	1	1

Table 2. Computational results of model P_1 and robust counterpart models P_3, P_4 and P_8 for OR-Library samples.

$\bar{f}_j$	Demand	Model Type / Information for Solving the Model	P_1	P_3	P_4	P_8 $\Gamma = 0,25$	$\Gamma = 0,5$	$\Gamma = 0,75$	$\Gamma = 1$
$0,1 * f_j$	507	OPT	177,0000	186,1500	--------	187,0500	187,1000	187,1500	187,2000
		CPU	0,4380	6,8910	--------	5,4060	5,6090	5,4210	5,4380
		ERROR	0,0000	0,0517	--------	0,0568	0,0570	0,0573	0,0576
		Number of facilities to cover demand points	116	120	--------	125	125	125	125
$0,1 * f_j$	582	OPT	218,0000	230,8750	--------	216,0500	216,1000	216,1500	216,2000
		CPU	0,4220	4,2500	--------	4,5150	4,0940	4,0780	4,1870
		ERROR	0,0000	0,0590	--------	0,0089	0,0087	0,0084	0,0082
		Number of facilities to cover demand points	164	165	--------	161	161	161	161
$0,01 * f_j$	507	OPT	177,0000	185,1200	--------	187,0050	187,0100	187,0150	187,0200
		CPU	0,4380	5,5150	--------	5,3750	5,3750	5,3590	5,6720
		ERROR	0,0000	0,0459	--------	0,0565	0,0565	0,0566	0,0566
		Number of facilities to cover demand points	116	122	--------	125	125	125	125
	582	OPT	218,0000	218,2420	--------	216,0050	216,0100	216,0150	216,0200
		CPU	0,4220	3,9690	--------	4,2190	4,2190	4,2500	4,0780
		ERROR	0,0000	0,0011	--------	0,0091	0,0091	0,0091	0,0091
		Number of facilities to cover demand points	161	161	--------	161	161	161	161

The results in *Tables 1* and *2* bring about the following conclusions. According to the results obtained of *Tables 1* and *2* these items are concluded.

I. As expected, the box approach is strongly conserving; therefore, its objective function value is more than those of the nominal model and the robust counterpart model with the adjustable conservatism approach. The box approach has more performance time and relative error with respect to other robust counterpart models. On average, the number of selected facilities in the box model is less than that of the nominal model.

II. The ellipsoidal model has an objective function closer to the nominal model. Furthermore, for the tentative samples with 50 and 100 demand points, the objective function value of the ellipsoidal approach is closer to the nominal model compared with the adjustable conservatism and box approaches. For random samples, the ellipsoid model results in better answers close to the nominal problem and consequently of less robustness with respect to other robust counterpart models. In addition, the ellipsoidal model, owing to its nonlinear form, does not present plausible answers for large size samples. The ellipsoidal approach has more CPU time and less relative error than the nominal problem compared with the other robust counterpart models compared to.

III. The optimized answers of the adjustable conservatism approach is closer to the objective function value of the nominal problem compared with the box approach. Moreover, for the tentative samples, the objective function value of the adjustable conservatism approach, is more distant from the nominal SCLP value by increasing the conserving level (increasing Γ). On average, by increasing the conserving level, the adjustable conservatism approach has more performance time and more relative error compared with other robust counterpart models.

IV. By decreasing the uncertainty interval from $0,1 * f_j$ to $0,01 * f_j$, the objective function values of the robust counterpart models approach those of the nominal model.

5.2 | Results of Multi-Period Set Covering Location Problem

In this section, the performance of nominal model P_9 is compared with the robust counterpart *Models (11)-(13)*. Furthermore, for a better comparison, the tentative samples and their parameters are considered similar to those for the single-period problems. The nominal value of the fixed establishment costs in four time periods is also assumed as $f_t = \alpha_t f$ where $\alpha_t = 0/5, 1/25, 1/75, 2/5$ in the four time periods, respectively. It means that the fixed establishment costs in the first period are half of their nominal value.

In the second period, the fixed establishment costs are 1.25 times of the primary value. The fixed establishment costs in the third and fourth periods change also to 1.75 and 2.5 times of the primary value. The nominal values of the fixed establishment costs are its exact and specified value. In this case, the fixed establishment costs are considered ascending and incremental. In the following, for a better comparison of the results, the fixed establishment costs of irregular changes over different time periods, i,e. $\alpha_t = 2/5,3/7,0/5,1/3$, are studied in addition to those of $\alpha_t = 1/8,2,2/2,2/4$.

Tables 3-5 present the variation of the uncertain fixed establishment parameters over time periods of $\alpha_t = 0/5,1/25,1/75,2/5$, $\alpha_t = 2/5,3/7,0/5,1/3$ and $\alpha_t = 1/8,2,2/2,2/4$, respectively. The first left column introduces the intended sample and the second one includes the information necessary for solving the model. To solve each model for the intended sample, as in *Table 1*, the optimum value of the model's objective function (OPT), solution time in second (CPU), its relative error compared to the nominal error (ERROR), the number of facilities covering demand points in each period and its robust counterpart models are presented here.

The third column shows the objective function value obtained by solving the nominal Problem (1). The fourth to seventh columns present the resulting variation obtained from the nominal models considering the uncertain fixed establishment cost parameters for time periods $\alpha_t = 0/5,1/25,1/75,2/5$ (*Table 3*), $\alpha_t = 2/5,3/7,0/5,1/3$ (*Table 4*) and $\alpha_t = 1/8,2,2/2,2/4$ (*Table 5*). The eighth, ninth and tenth columns exhibit the objective function values of the nominal Problem (9), robust counterpart *Model (11)* and the robust counterpart *Model (12)*, respectively.

The eleventh to fourteenth columns present the objective function value obtained by solving the multi-period problem based on the adjustable conservatism approach with conserving level of $\Gamma = 0.25,0.5,0.75,1$, respectively. As for the single-period model, GAMS software is unable to perform the ellipsoidal model calculations for ORL-ibrary real data. The random samples applied in *Table 1* are used to study the solution process of the robust counterpart models and also the accuracy of the results.

Table 3. Results obtained by solving models P_1, P_9 and robust counterpart models P_{11}, P_{12} and P_{13} for $\alpha_t = 0/5, 1/25, 1/75, 2/5$ time periods based on random samples and OR-Library data.

Demand	Model Type / Information for Solving the Model	P_1	Results of Solving Exact Models P_1 with $f_t = \alpha_t f$				P_9	P_{11}	P_{12}	P_{13}			
			$\alpha_1 = 0/5$	$\alpha_2 = 1/25$	$\alpha_3 = 1/75$	$\alpha_4 = 2/5$				$\Gamma = 0,25$	$\Gamma = 0,5$	$\Gamma = 0,75$	$\Gamma = 1$
50	OPT	133,0000	133,0000	133,0000	266,0000	399,0000	266,0000	332,5000	277,9740	278,5000	291,0000	303,5000	316,0000
	CPU	0,0000	0,0000	0,0000	0,7190	0,0160	0,0160	0,0150	0,0160	0,0160	0,0150	0,2660	0,1560
	ERROR	0,0000	0,5000	0,5000	0,0000	0,5000	0,0000	0,2500	0,0450	0,0470	0,0940	0,1410	0,1880
	Number of facilities to cover demand points	4	4	4	4	4	4	4	4	4	4	4	4
100	OPT	34,0000	34,0000	34,0000	68,0000	102,0000	68,0000	85,0000	73,8390	76,5000	85,0000	93,5000	102,0000
	CPU	0,0310	1,7190	1,7190	0,0160	0,0630	1,7190	1,7190	0,0160	0,0000	0,0160	0,0160	0,0180
	ERROR	0,0000	0,5000	0,5000	0,0000	0,5000	0,0000	0,2500	0,0859	0,1250	0,2500	0,3750	0,5000
	Number of facilities to cover demand points	1	1	1	1	1	1	1	1	1	1	1	1
507	OPT	177,0000	177,0000	177,0000	354,0000	558,0000	295,0000	394,5000	-----------	298,5000	299,0000	299,5000	300,0000
	CPU	0,4380	0,6720	0,6410	0,4370	1,3290	0,5160	0,6100	-----------	0,9380	1,0150	1,1870	1,9690
	ERROR	0,0000	0,4000	0,4000	0,2000	0,8915	0,0000	0,3373	-----------	0,0119	0,0135	0,0152	0,0169
	Number of facilities to cover demand points	116	116	116	116	123	108	111	-----------	109	109	109	109
582	OPT	218,0000	218,0000	218,0000	436,0000	648,0000	410,0000	556,0000	-----------	414,5000	415,0000	415,5000	420,0000
	CPU	0,4220	0,5000	1,4690	0,4680	0,4370	0,5620	0,4530	-----------	0,7810	0,7960	0,7660	0,8130
	ERROR	0,0000	0,4682	0,4682	0,0634	0,5805	0,0000	0,3561	-----------	0,0110	0,0122	0,0134	0,0244
	Number of facilities to cover demand points	164	163	163	163	160	159	158	-----------	162	162	162	163

Table 4. Results obtained by solving models P_1, P_9 and robust counterpart models P_{11}, P_{12} and P_{13} for $\alpha_t = 2/5, 3/7, 0/5, 1/3$ time periods based on random samples and OR-Library data.

Demand	Model Type	P_1	Results of Solving Exact Models P_1 with $f_t = \alpha_t f$				P_9	P_{11}	P_{12}	P_{13}			
	Information for Solving the Model		$\alpha_1 = 2/5$	$\alpha_2 = 3/7$	$\alpha_3 = 0/5$	$\alpha_4 = 1/3$				$\Gamma = 0,25$	$\Gamma = 0,5$	$\Gamma = 0,75$	$\Gamma = 1$
50	OPT	133,0000	399,0000	532,0000	133,0000	133,0000	333,0000	433,0000	350,9900	351,7500	370,5000	389,2500	408,0000
	CPU	0,0000	0,0000	0,0000	0,0150	0,0150	0,0470	0,0470	0,0940	0,0150	0,0000	0,0000	0,0160
	ERROR	0,0000	0,1982	0,5976	0,6006	0,6006	0,0000	0,3003	0,0540	0,0563	0,1126	0,1689	0,2252
	number of facilities to cover demand points	4	4	4	4	4	4	4	4	4	4	4	4
100	OPT	34,0000	102,0000	136,0000	34,0000	34,0000	85,0000	110,5000	93,7590	97,7500	110,5000	123,2500	136,0000
	CPU	0,0310	0,0000	0,0160	0,0160	0,0160	0,1090	0,0160	0,1570	0,0000	0,0160	0,0160	0,0150
	ERROR	0,0000	0,2000	0,6000	0,6000	0,6000	0,0000	0,3000	0,1030	0,1500	0,3000	0,4500	0,6000
	number of facilities to cover demand points	1	1	1	1	1	1	1	1	1	1	1	1
507	OPT	177,0000	558,0000	708,0000	177,0000	177,0000	483,0000	632,5000	-----------	488,7500	489,5000	490,2500	491,0000
	CPU	0,4380	0,4530	0,5780	0,4840	0,5460	0,6410	0,4530	-----------	0,8600	0,8750	0,8440	0,8280
	ERROR	0,0000	0,1553	0,4658	0,6335	0,6335	0,0000	0,3095	-----------	0,0119	0,0134	0,0150	0,0166
	number of facilities to cover demand points	116	123	116	116	116	113	110	-----------	112	112	112	112
582	OPT	218,0000	648,0000	868,0000	218,0000	218,0000	605,00•0	778,0000	-----------	601,7500	602,5000	603,2500	619,0000
	CPU	0,4220	0,437	0,4370	0,4220	0,4370	0,4370	0,4690	-----------	0,7970	0,7810	1,1090	0,8440
	ERROR	0,0000	0,0711	0,4347	0,6397	0,6397	0,0000	0,2860	-----------	0,0054	0,0041	0,0029	0,0231
	Number of facilities to cover demand points	164	160	162	163	163	149	147	-----------	149	149	149	152

Table 5. Results obtained by solving models P_1, P_9 and robust counterpart models P_{11}, P_{12} and P_{13} for $\alpha_t = 1/8, 2, 2/2, 2/4$ time periods based on random samples and OR-Library data.

Demand	Model Type	P_1	Results Of Solving Exact Models P_1 with $f_t = \alpha_t f$				P_9	P_{11}	P_{12}	P_{13}			
	Information for Solving the Model		$\alpha_1 = 1/8$	$\alpha_2 = 2$	$\alpha_3 = 2/2$	$\alpha_4 = 2/4$				$\Gamma = 0,25$	$\Gamma = 0,5$	$\Gamma = 0,75$	$\Gamma = 1$
50	OPT	133,0000	239,4000	266,0000	292,6000	319,2000	279,3000	299,2500	282,8920	283,0500	286,8000	290,5500	294,3000
	CPU	0,0000	0,5620	0,0000	0,0470	0,5930	0,0160	0,0160	0,0150	0,0150	0,0160	0,0470	0,0160
	ERROR	0,0000	0,1429	0,0476	0,0476	0,1429	0,0000	0,0714	0,0547	0,0134	0,0268	0,0403	0,0537
	Number of facilities to cover demand points	4	4	4	4	4	4	4	4	4	4	4	4
100	OPT	34,0000	61,2000	68,0000	74,8000	81,6000	71,4000	76,5000	73,1520	73,9500	76,5000	79,0500	81,6000
	CPU	0,0310	0,4380	0,0150	0,0160	0,0150	0,0160	0,0000	0,0150	0,0000	0,0150	0,0160	0,0150
	ERROR	0,0000	0,1429	0,0476	0,0476	0,1429	0,0000	0,0714	0,0245	0,0357	0,0714	0,1071	0,1428
	Number of facilities to cover demand points	1	1	1	1	1	1	1	1	1	1	1	1
507	OPT	177,0000	329,4000	354,0000	400,4000	453,6000	386,4000	409,5000	-----------	386,5500	386,7000	386,8500	387,0000
	CPU	0,4380	0,4530	0,4530	0,4680	0,5460	0,4380	0,4840	-----------	1,2810	0,8910	0,9210	1,6250
	ERROR	0,0000	0,1475	0,0838	0,0362	0,1739	0,0000	0,0598	-----------	0,00039	0,00078	0,0012	0,0015
	Number of facilities to cover demand points	116	119	116	120	124	120	119	-----------	123	123	123	123
582	OPT	218,0000	388,8000	436,0000	475,2000	518,4000	457,8000	488,2500	-----------	451,6500	451,8000	451,9500	454,2000
	CPU	0,4220	0,4370	1,3440	0,4380	1,0160	0,5930	0,4530	-----------	0,8750	1,7660	0,8130	0,8120
	ERROR	0,0000	0,1507	0,01476	0,0380	0,1324	0,0000	0,0665	-----------	0,0134	0,0131	0,0127	0,0079
	Number of facilities to cover demand points	164	162	163	163	161	162	162	-----------	161	161	161	161

The first three results of single-period models are similarly established here. In particular, for the multi-period problems, the data in *Tables 3*, *4* and *5* result in the following conclusions:

I. The values of the objective function model of the nominal problem for various time periods are close to that of the period with least variations. Since the objective function values for smaller time periods are rather similar, so the optimum answer in a smaller time period is acceptable.

II. Furthermore, the multi-period nominal problem with $\alpha_t = 0/5,1/25,1/75,2/5$ time period, has a smaller uncertain interval compared to $\alpha_t = 2/5,3/7,0/5,1/3$. In addition, since each uncertain interval is considered symmetrical in $\alpha_t = 1/8,2,2/2,2/4$ time period, the objective function value of the multi-period nominal problem is closer to the middle period.

6 | Conclusion

Concerning the uncertainty in parameters, applying nominal models may result in unreliable answers. To deal with this problem and to obtain a more realistic model, a robust optimization approach may be useful. In this research, at first, an uncertain SCLP under uncertainty in the fixed establishment costs was studied with the robust optimization approach. Then, this problem was considered dynamically over various time periods. Furthermore, these multi-period and single-period nominal models were compared with the robust counterpart models based on the box approach, ellipsoidal approach and the adjustable conservatism one.

According to the obtained results, the box approach is strongly conserving and its objective function value is farther from that of the nominal models compared to those of the adjustable conservatism adjustment and ellipsoidal approaches. Also, it was observed that, the objective function value of the adjustable conservatism approach with more conserving level is more distant from that of the nominal problem. Among the three robust optimization approaches, the ellipsoidal one has closer objective function value to that of the nominal problem model with respect to the box and adjustable conservatism approaches. The multi-period problems were explored over various time periods and it was observed that their order being ascending, descending and irregular has no effect on the selection of uncertain interval. So, the objective function value of the nominal problems in uncertain intervals is closer to the middle period. In order to complete the models proposed in this research, one may consider other uncertain parameters. In addition, for the proposed models of this research getting closer to the real world situations, one can consider some other parameters such as opening and closing cost of facilities during various time periods or maintenance cost of facilities. As another proposal, the robust counterpart models may be solved by other exact or approximate methods for optimization problems calculations.

Conflict of Interest

The authors hereby declare that there is no conflict of interest of any kind that could have influenced the design, execution, analysis, interpretation, or reporting of the results presented in this study. This research was conducted independently and without receiving any direct or indirect financial support, sponsorship, or funding from commercial, industrial, governmental, or private organizations that could be perceived as having a potential interest in the outcomes of this work.

All stages of model formulation, development of robust optimization approaches—including the box, ellipsoidal, and conservatism by adjustment methods—computational implementation, and analysis were performed solely for academic and scientific purposes and in accordance with accepted standards of research integrity. The OR-library benchmark datasets were used exclusively to evaluate the performance of the proposed robust counterparts of the single-period and multi-period set covering location problems, to validate the models, and to ensure the reproducibility and comparability of the results. The authors have no financial or organizational affiliation with the providers of these datasets.

Authors' Contributions

Furthermore, the authors confirm that there are no personal, professional, or institutional relationships, including consultancies, employment, shareholding, or other financial interests, that could potentially result in bias in the selection of methods, interpretation of findings, or conclusions of this paper. Full responsibility for the content of this article, including the accuracy of the data, models, and analyses, is solely borne by the authors. The purpose of this research is to contribute to the advancement of knowledge in the field of covering location problems under uncertainty and to provide robust decision-making tools for real-world applications.

References

[1] Toregas, C., Swain, R., ReVelle, C., & Bergman, L. (1971). The location of emergency service facilities. *Operations research*, *19*(6), 1363–1373. https://doi.org/10.1287/opre.19.6.1363

[2] Karp, R. M. (2009). Reducibility among combinatorial problems. In *50 years of integer programming 1958-2008: From the early years to the state-of-the-art* (pp. 219–241). Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-68279-0_8

[3] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

[4] Eaton, D. J., Daskin, M. S., Simmons, D., Bulloch, B., & Jansma, G. (1985). Determining emergency medical service vehicle deployment in Austin, Texas. *Interfaces*, *15*(1), 96-108. https://doi.org/10.1287/inte.15.1.96

[5] Ceria, S., Nobili, P., & Sassano, A. (1998). A lagrangian-based heuristic for large-scale set covering problems. *Mathematical programming*, *81*(2), 215-281. https://doi.org/10.1007/BF01581106

[6] Desrochers, M., Dumas, Y., Soumis, F., & Trudeau, P. (1991). Column generation approaches to the airline crew scheduling problems. *TRISTAN i (triennial symposium on transportation analysis)* (pp. 1-133). TRISTAN I Program. https://tristanconference.org/system/tristan-i/documents/TRISTAN-I-program.pdf#page=20

[7] Ahmed, Sh., & Papageorgiou, D. J. (2013). Probabilistic set covering with correlations. *Operations research*, *61*(2), 438-452. https://doi.org/10.1287/opre.1120.1135

[8] Zanjirani Farahani, R., & Hekmatfar, M. (2009). *Facility location: Concepts, models, algorithms and case studies*. Physica-Verlag Heidelber(Springer). https://doi.org/10.1007/978-3-7908-2151-2

[9] Dantzig, G. B. (1955). Linear programming under uncertainty. *Management science*, *1*(3), 197-203. https://doi.org/10.1287/mnsc.1.3-4.197

[10] Deng, J. (1982). Control problems of grey systems. *Systems & control letters*, *1*(5), 288–294. https://doi.org/10.1016/S0167-6911(82)80025-X

[11] Beale, E. M. L., & Dantzig, G. B. (1955). On minimizing a convex function subject to linear inequalities. *Journal of the royal statistical society: Series b (methodological)*, *17*(2), 173-184. https://doi.org/10.1111/j.2517-6161.1955.tb00191.x

[12] Cheng, Y., Chen, X., Li, H., Cheng, Z., Jiang, R., Lu, J., & Fu, H. (2018). Analysis and comparison of bayesian methods for measurement uncertainty evaluation. *Mathematical problems in engineering*, 7509046. https://doi.org/10.1155/2018/7509046

[13] Liu, B. (2007). *Uncertainty theory*. Springer Berlin Heidelberg (Springer-Verlag). https://doi.org/10.1007/978-3-540-73165-8

[14] Soyster, A. L. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations research*, *21*(5), 1154-1157. https://doi.org/10.1287/opre.21.5.1154

[15] Beraldi, P., & Ruszczyński, A. (2002). The probabilistic set-covering problem. *Operations research*, *50*(6), 956-967. https://doi.org/10.1287/opre.50.6.956.345

[16] Hwang, H. S. (2004). A stochastic set-covering location model for both ameliorating and deteriorating items. *Computers and industrial engineering*, *46*(2), 313-319. https://doi.org/10.1016/j.cie.2003.12.010

[17] Chiang, C. I., Hwang, M. J., & Liu, Y. H. (2005). An alternative formulation for certain fuzzy set-covering problems. *Mathematical and computer modelling*, *42*(4-3), 363-365. https://doi.org/10.1016/j.mcm.2004.05.012

[18] Saxena, A., Goyal, V., & Lejeune, M. A. (2010). MIP reformulations of the probabilistic set covering problem. *Mathematical programming*, *121*(1), 1-31. https://doi.org/10.1007/s10107-008-0224-y

[19] Gunawardane, G. (1982). Dynamic versions of set covering type public facility location problems. *European journal of operational research*, *10*(2), 190-195. https://doi.org/10.1016/0377-2217(82)90159-X

[20] Moradi, M., Salahi, M., Bardsiri, M., & Jamalian, A. (2014). A new model based on supply chain network design under uncertainty. *Journal of operations research in its applications, 11*(2), 26-29. (**In Persian**). http://jamlu.liau.ac.ir/article-1-821-fa.html

[21] Ben-Tal, A., El-Ghaoui, L., & Nemirovski, A. (2009). *Robust optimization*. Springer. https://www.torrossa.com/en/resources/an/5576051

[22] Bertsimas, D., & Sim, M. (2004). The price of robustness. *Operations research*, *52*(1), 35-53. https://doi.org/10.1287/opre.1030.0065

[23] Inuiguchi, M., & Sakawa, M. (1998). Robust optimization under softness in a fuzzy linear programming problem. *International journal of approximate reasoning*, *18*(1-2), 21-34. https://doi.org/10.1016/S0888-613X(97)10002-0

[24] Mulvey, J. M., Vanderbei, R. J., & Zenios, S. A. (1995). Robust optimization of large-scale systems. *Operations research*, *43*(2), 264-281. https://doi.org/10.1287/opre.43.2.264

[25] Pan, F., & Nagi, R. (2010). Robust supply chain design under uncertain demand in agile manufacturing. *Computers & operations research*, *37*(4), 668-683. https://doi.org/10.1016/j.cor.2009.06.017

[26] Rezvani, S., Moghadas, M., & Zanjirani Farahani, D. (2019). *The dynamic covering location problem with uncertainty* [Thesis]. (**In Persian**). https://ganj.irandoc.ac.ir/#/articles/59b73a8999393ef0664a8ecaf73369b2

[27] Pereira, J., & Averbakh, I. (2013). The robust set covering problem with interval data. *Annals of operations research*, *207*(1), 217-235. https://doi.org/10.1007/s10479-011-0876-5%0A%0A

[28] Lutter, P., Degel, D., Büsing, C., Koster, A. M. C. A., & Werners, B. (2017). Improved handling of uncertainty and robustness in set covering problems. *European journal of operational research*, *263*(1), 35-49. https://doi.org/10.1016/j.ejor.2017.04.044

[29] Ben-Tal, A., & Nemirovski, N. (1998). Robust convex optimization. *Mathematics of operations research, 23*(4), 769-805. https://doi.org/10.1287/moor.23.4.769

[30] El Ghaoui, L., Lebret, H. (1997). Robust solutions to least-squares problems to uncertain data matrices. *SIAM journal on matrix analysis and applications*, *18*(4), 1035-1064. https://doi.org/10.1137/S0895479896298130