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Standardized Combined Efficiency in a Two-Stage Data Envelopment Analysis Network

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Abstract

Optimizing systems based on decision-making criteria across multiple sets requires selecting different stages to achieve the best efficiency for the analyst's objective. Therefore, advantages and disadvantages must be considered simultaneously to choose the most effective method. The Hurwicz criterion is an approach that combines pessimistic and optimistic criteria to achieve optimal efficiency. This method allows for solving more complex problems in two or multiple stages. In the standardized combined approach within a two-stage Data Envelopment Analysis (DEA) network, the outputs of the first stage are selected as the inputs of the intermediate stage, ultimately determining the Most Productive Scale Size (MPSS). By applying the Hurwicz method in both optimistic and pessimistic scenarios, the best Decision-Making Units (DMUs) are selected for analysis.

Keywords: Data envelopment analysis, Optimistic, Pessimistic, Hurwicz criterion, Two-stage network.

1 | Introduction

Data Envelopment Analysis (DEA) is a method that uses mathematical programming to calculate the relative efficiency of Decision-Making Units (DMUs) with multiple inputs and outputs. It was first introduced by Charnes et al. [1], and Banker et al. [2] constructed a link between DEA, production estimation, and efficiency frontiers based on empirical observations. The CCR model is one of the most well-known DEA models, initially introduced by Charnes et. al [1] to measure the efficiency of a set of DMUs. This model extends Farrell's efficiency measurement to a multi-input, multi-output framework and calculates radial efficiency under Constant Returns to Scale (CRS). The CCR model has two orientations: Input-oriented (Also known as the envelopment form) and output-oriented (Also known as the multiplier form). In DEA, returns to scale refer to the highest level of productivity, which is considered the efficient unit. Moreover, the highest productivity level, or the efficient unit, is closely related to the concept of CRS introduced by Banker [2] in

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DEA. Banker et al. [3] further explored the highest level of productivity as part of the literature on returns to scale. However, all these scientific studies have evaluated DEA from an optimistic point of view. The performance of DMUs can also be assessed from a pessimistic point of view, which may be a more interesting category. Studies on pessimistic efficiency measurement in DEA can be found in the research of Wang et al. [4], and Wang and Lan [5]. Chen et al. [6] analyzed efficiency scores to predict the efficiency frontier of a two-stage network DEA model. They concluded that the discrepancy between the efficiency frontier produced by the base model and the expected frontier produced is due to synergy effects with Variable Returns to Scale (VRS) across different stages.

Terry et al. [7] proposed a two-stage network for evaluating transit services in sub-Saharan African cities. In the first stage, they calculated efficiency and effectiveness scores with a specific bias. In the second stage, they adjusted efficiency scores based on bias correction against service and operational indices. Their findings indicate that units with higher capacity, in terms of vehicle kilometers traveled and daily ridership, are more efficient than units with lower capacity. Zhang et al. [8] investigated exogenous constraints on two-stage DEA models. They found that a fixed-sum output constraint, as an exogenous factor, affects efficiency scores in cooperative game theory based on centralized models and subjective global models they presented. Dar et al. [9] evaluated the efficiency and determinants of public health in India using a two-stage network DEA approach. They found that more than one-third of public healthcare services in India are inefficient. Their study concluded that socio-economic factors impact healthcare efficiency more than medical factors in India.

Since efficient units' performance may vary depending on different evaluation perspectives, applying double frontiers and the Hurwicz criterion allows for a more comprehensive assessment of each unit. Double frontiers efficiently capture both the optimistic and pessimistic performances of a decision-making unit, making it a more holistic approach than traditional methods.

In Section 2, the Hurwicz method is briefly discussed. Section 3 introduces a proposed approach combining a two-stage DEA network with the Hurwicz method to develop standardized combined efficiency in a two-stage DEA network. Finally, Section 4 presents a ranking of DMUs through a practical example.

2 | Hurwicz Method

The Hurwicz rule is a method applied in decision-making processes under conditions of uncertainty. This uncertainty arises from the fact that predicting the future accurately is often impossible. While individuals can forecast various phenomena and events, in most cases, it is too difficult to estimate the exact values of certain parameters such as temperature, company profits, the volume of finished products, product demand, prices, production costs, etc. which is highly challenging. If these data points are known, selecting the best option, such as the most profitable investment strategy, becomes straightforward. However, when many future factors remain uncertain at the time of decision-making, the decision-maker must choose the most suitable option based on expert judgment [10]. The Hurwicz method incorporates both optimistic and pessimistic points of view, typically leading to reasonable conclusions. Hurwicz [10] and [11] argue that a decision-maker should rank options $(I_j; j = 1, \dots, n)$ according to a weighted average of optimism and confidence levels. The Hurwicz criterion is defined as follows:

$$h_i = \lambda w_i + (1-\lambda) m_i, \qquad \text{for all j,} \tag{1}$$

where,

λ is the optimism coefficient, where $\lambda \in [0,1]$.

m_j And w_j represent the best and worst possible outcomes for each option, respectively.

The most well-known Hurwicz criterion, proposed by Hurwicz [10], selects the activities with the highest and lowest benefits for a given decision X and then assigns the following index to each activity:

$$\lambda \max(x) + (1-\lambda) \min(x)\,. \tag{2}$$

Thus, the activity (Or decision) with the highest index is preferable.

In the Hurwicz criterion, the λ parameter, which reflects the manager's level of optimism, is determined by the decision-maker. Since different managers have varying evaluation criteria, it is difficult to assign a universal value λ. By adjusting λ, the Hurwicz criterion can transform into different decision-making rules. For example:

I. When $\lambda = 0$ the pessimistic criterion is obtained.

II. When $\lambda = 1$ the optimistic criterion is applied.

There are several variations of the Hurwicz criterion.

By using the Hurwicz criterion to integrate pessimistic and optimistic efficiency measurements, standardized combined efficiency is calculated as follows:

$$\xi = (1-\lambda)\frac{\theta_j^{pes}}{\max\limits_{1\le j\le n}(\theta_j^{pes}} + \lambda\theta_j^{opt}, \tag{3}$$

where,

I. θ_j^{opt} is the optimistic efficiency.

II. θ_j^{pes} it is the pessimistic efficiency.

III. ξ_j represents the standardized combined efficiency.

These values are derived using the double frontiers approach for DMUs DMU_j. DMUs can follow a two-stage structure, where all outputs from the first stage are used as inputs for the second stage. The outputs of the first stage, in this case, are called intermediate measures (indices) in the second stage [12].

3 | Standardized Composite Efficiency in a Two-Stage Data Envelopment Analysis Network

Consider a network as shown in *Fig. 1* with $X_j = (x_{1j},...,x_{mj})$ input indices, $Z_j = (z_{1j},...,z_{kj})$ intermediate indices (Outputs of the first stage that serve as inputs for the second stage), and $Y_j = (y_{1j},...,y_{sj})$ final output indices.

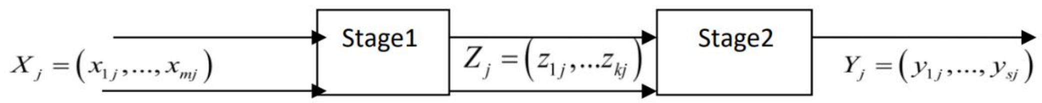

Fig. 1. Two-stage process.

The efficiency of the first and second stages, assuming VRS, is defined as follows:

$$e_j^{(1)} = \frac{\sum_{k=1}^{k} w_k z_{kj} + U_0}{\sum_{i=1}^{m} v_i x_{ij}}, e_j^{(2)} = \frac{\sum_{r=1}^{s} u_r y_{rj} + \bar{U}_0}{\sum_{K=1}^{K} W_K Z_{kj}}, \quad j = 1, ...n. \tag{4}$$

To calculate overall efficiency, we consider two stages:

We maximize the efficiency of the first stage for the decision-making unit under evaluation (DMU_p), ensuring that the efficiency of both stages for all units remains less than or equal to one. This is referred to as "optimistic leader efficiency." Since cost control (Input) is more feasible than profit control (Output), the first stage is called the leader, and the second stage is the follower [13]. Therefore, we use *Model (5)*:

$$\theta_p^{*opt(1)} = \text{Max}\frac{\sum w_{k=1}^{K} w_k z_{kp} + U_0}{\sum W_{i=1}^{m} v_i x_{ip}},$$
$$\text{s.t} \quad \frac{\sum w_{k=1}^{K} w_k z_{kj} + U_0}{\sum w_{i=1}^{m} v_i x_{ij}} \leq 1, j = 1, \dots, n,$$
$$\frac{\sum w_{k=1}^{K} u_r y_{kj} + \bar{U}_0}{\sum w_{k=1}^{K} w_k z_{kj}} \leq 1, j = 1, \dots, n, \tag{5}$$
$$v_i \geq 0, w_k \geq 0, u_r \geq 0, \text{for all i, for all k, for all r.}$$

Since *Model (5)* is nonlinear, we linearize it as follows:

$$\sum_{i=1}^{m} v_i x_{ip} = \frac{1}{t} \quad \text{and } v_i = tv_i,\ w_k = tw_k, u_r = tu_r,$$
$$u_r \geq \varepsilon, w_k \geq \varepsilon, v_i \geq \varepsilon; \text{for all r, for all k, for all i.} \tag{6}$$

Thus, we obtain:

$$\theta_p^{*opt(1)} = \text{Max} \sum_{k=1}^{K} w_k z_{kp} + U_0,$$
$$\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{ip} = 1,$$
$$\sum_{k=1}^{K} w_k z_{kj} - \sum w_{i=1}^{m} v_i x_{ij} + U_0 \leq 0, j = 1, \dots, n, \tag{7}$$
$$\sum_{k=1}^{K} u_r y_{rj} - \sum w_{k=1}^{K} w_k z_{kj} + \bar{U}_0 \leq 0, j = 1, \dots, n,$$
$$v_i \geq 0, w_k \geq 0, u_r \geq 0, \text{for all i, for all k, for all r,}$$

where $\theta_p^{*opt(1)}$ represents the optimistic efficiency of stage one DMU_p. To compute the pessimistic efficiency of the first stage, assuming the VRS, we use the following:

$$\theta_p^{*pec(1)} = \text{Min}\frac{\sum_{k=1}^{K} w_k z_{kp} + U_0}{\sum_{i=1}^{m} v_i x_{ip}}$$
$$\text{s.t.} \quad \frac{\sum_{k=1}^{K} w_k z_{kj} + U_0}{\sum_{i=1}^{m} v_i x_{ij}} \geq 1, j = 1, \dots, n,$$
$$\frac{\sum_{k=1}^{K} u_r y_{rj} + \bar{U}_0}{\sum_{k=1}^{K} w_k z_{kj}} \geq 1, j = 1, \dots, n, \tag{8}$$
$$v_i \geq 0, w_k \geq 0, u_r \geq 0, \text{for all i, for all k, for all r.}$$

By applying the variable transformation in *Eq. (6)*, we obtain the linear form:

$$\theta_p^{*pec(1)=\text{Min}} \sum_{k=1}^{K} w_k z_{kp} + U_0,$$
$$\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{ip} = 1,$$
$$\sum_{k=1}^{K} w_k z_{kj} - \sum W_{i=1}^{m} v_i x_{ij} + U_0 \geq 0, j = 1, \dots, n, \tag{9}$$
$$\sum_{k=1}^{K} u_r y_{rj} - \sum W_{k=1}^{K} w_k z_{kj} + \bar{U}_0 \geq 0, j = 1, \dots, n,$$
$$v_i \geq 0, w_k \geq 0, u_r \geq 0, \text{for all i, for all k, for all r,}$$

In the second stage (Follower), to compute the optimistic efficiency of under-VRS, we use the following model:

$$\begin{aligned}
\theta_p^{*opt(2)} = \text{Max} & \frac{\sum_{r=1}^{s} u_r y_{rp} + \bar{U}_0}{\sum_{k=1}^{K} w_k z_{kp}}, \\
\text{s.t.} \quad & \frac{\sum_{k=1}^{K} w_k z_{kp} + U_0}{\sum_{i=1}^{m} v_i x_{ip}} = \theta_p^{*pes(1)}, \\
& \frac{\sum_{k=1}^{K} w_k z_{kj} + U_0}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, j = 1, \dots, n, \\
& \frac{\sum_{r=1}^{s} u_r y_{rj} + \bar{U}_0}{\sum_{k=1}^{K} w_k z_{kj}} \leq 1, j = 1, \dots, n, \\
& u_r \geq \varepsilon, w_k \geq \varepsilon, v_i \geq \varepsilon; \text{for all r, for all k, for all i,}
\end{aligned} \tag{10}$$

Using the variable transformation:

$$\begin{aligned}
& \sum_{k=1}^{K} w_k z_{kp} = \frac{1}{\alpha}, \\
& W_k = \alpha w_k, \\
& V_i = \alpha v_i, U_r = \alpha u_r, \\
& (K = 1,\dots,k, r = 1,\dots,s, i = 1,\dots,m).
\end{aligned} \tag{11}$$

The linear form of *Model (10)* becomes:

$$\begin{aligned}
\theta_p^{*opt(2)} = \text{Max} \quad & \sum_{r=1}^{s} u_r y_{rp} + \bar{U}_0, \\
\text{s.t.} \quad & \sum_{k=1}^{K} w_k z_{kp} = 1, \\
& \sum_{k=1}^{K} w_k z_{kp} + U_0 = \theta_p^{*opt(1)} . \sum_{i=1}^{m} v_i x_{ip}, \\
& \sum_{k=1}^{K} w_k z_{kj} - \sum_{i=1}^{m} v_i x_{ij} + U_0 \leq 0, j = 1,\dots,n, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{k=1}^{K} w_k z_{kj} + \bar{U}_0 \leq 0, j = 1,\dots,n, \\
& u_r \geq \varepsilon, w_k \geq \varepsilon, v_i \geq \varepsilon, \text{for all r, for all k, for all i,}
\end{aligned} \tag{12}$$

where $\theta_p^{*opt(2)}$ represents the optimistic efficiency of stage two (Follower). To compute the pessimistic efficiency of the second stage, we use:

$$\begin{aligned}
\theta_p^{*pes(2)} = \text{Min} & \frac{\sum_{r=1}^{s} u_r y_{rp} + \bar{U}_0}{\sum_{k=1}^{K} w_k z_{kp}}, \\
\text{s.t.} & \frac{\sum_{k=1}^{K} w_k z_{kp} + U_0}{\sum_{i=1}^{m} v_i x_{ip}} = \theta_p^{*pes(1)}, \\
& \frac{\sum_{k=1}^{K} w_k z_{kj} + U_0}{\sum_{i=1}^{m} v_i x_{ij}} \geq 1; j = 1, \dots, n, \\
& \frac{\sum_{r=1}^{s} u_r y_{rj} + \bar{U}_0}{\sum_{k=1}^{K} w_k z_{kj}} \geq 1; j = 1, \dots, n, \\
& u_r \geq \varepsilon, w_k \geq \varepsilon, v_i \geq \varepsilon; \text{for all r, for all k, for all i.}
\end{aligned} \tag{13}$$

Applying the variable *Model (11)*, we obtain the linear model:

$$\theta_p^{*pec(2)} = \text{Min} \sum_{r=1}^{s} u_r y_{rp} + U_0,$$
$$\text{s.t.} \sum_{k=1}^{k} w_k z_{kp} = 1,$$
$$\sum_{k=1}^{K} w_k z_{kp} + U_0 = \theta_p^{*pec(1)}.\sum_{i=1}^{m} v_i x_{ip} \geq 0, j = 1, \dots, n, \tag{14}$$
$$\sum_{k=1}^{K} u_r y_{rj} - \sum_{k=1}^{K} w \; w_k z_{kj} + \bar{U}_0 \geq 0, j = 1, \dots, n,$$
$$u_r \geq \varepsilon, w_k \geq \varepsilon, v_i \geq \varepsilon, \text{for all i, for all k, for all r,}$$

where $\theta_p^{*pes(2)}$ represents the pessimistic efficiency of the second stage (Follower).

To integrate the optimistic and pessimistic efficiency measures of both stages, we use the standardized combined efficiency based on the Hurwicz criterion:

$$\xi_j^{(h)} = (1-\lambda)\frac{\theta^{*pes(h)}}{\max\theta_j^{*pes(h)}} + \lambda\theta_j^{*opt(h)}, h = 1,2, \tag{15}$$

where $\xi_j^{(1)}$ $\xi_j^{(2)}$ and represent the standardized combined efficiencies for stage one and stage two, respectively $0 \leq \lambda \leq 1$. The overall standardized combined efficiency is defined as:

$$\xi_j = \Gamma_1\xi_j^{(1)} + \Gamma_2\xi_j^{(2)}, \Gamma_1 + \Gamma_2 = 1, \Gamma_1, \Gamma_2 \epsilon [0,1], \tag{16}$$

where, $\xi_j^{(1)}$ The standardized combined efficiency is derived from the double frontiers approach for stage one (Leader) DMU_j.

$\xi_j^{(2)}$ Represents the standardized combined efficiency derived from the double frontiers approach for stage two (Follower) DMU_j.

ξ_j Represents the overall standardized combined efficiency assuming VRS of DMU_j.

4 | Application Example

In this section, we examine the models presented in the previous section. The data pertains to 27 companies, which are artificial but close to reality and generated using artificial intelligence. These data correspond to a two-stage network with two input variables, two intermediate variables, and one output variable.

The studied indicators are as follows:

First-stage input indicators:

I. x_{1j}; (Working hours: Number of work hours).

II. x_{2j}; (Operational costs: Wages, licensing fees, minor repairs, administrative expenses, travel and distribution costs, commissions, leasing, raw materials, insurance, and property taxes).

Intermediate indicators:

I. z_{1j}; processed products (Number of completed products).

II. z_{2j}; service requests fulfilled (Number of service requests handled).

Second-stage output indicator:

y_{1j}; final output (Goods or services sold in the market, measured in monetary value).

The values of these indicators are presented in *Table 1*.

Table 1. Index values.

DMU_j	x_{1j}	x_{2j}	z_{1j}	z_{2j}	y_{1j}
1	115	2600	84	52	6700
2	120	3100	87	54	4200
3	125	3600	90	56	4700
4	130	4100	93	44	5200
5	135	2100	96	46	5700
6	140	2600	75	48	6200
7	145	3100	78	50	6700
8	150	3600	81	52	4200
9	155	4100	84	54	4700
10	120	2200	92	60	5400
11	125	2700	95	48	5900
12	130	3200	98	50	6400
13	135	3700	101	52	6900
14	140	4200	80	54	4400
15	145	2200	83	56	4900
16	150	2700	86	58	5400
17	155	3200	89	60	5900
18	160	3700	92	48	6400
19	165	4200	95	50	6900
20	130	2300	103	56	4600
21	135	2800	106	58	5100
22	140	3300	85	60	5600
23	145	3800	88	62	6100
24	150	4300	91	64	6600
25	155	2300	94	52	7100
26	160	2800	97	54	4600
27	165	3300	100	56	5100

Applying the proposed models, the optimistic efficiency (Leader) in the optimistic state and the pessimistic efficiency (Follower) in the pessimistic state are calculated for the first and second stages of the units. These values are presented in *Table 2*.

Table 2. Model execution results and unit rankings.

DMU_p	$\theta_p^{*opt(1)}$	$\theta_p^{*opt(1)}$	$\theta_p^{*pec(1)}$	$\theta_p^{*pes(1)}$	$\xi_j^{(1)}$	$\xi_j^{(2)}$	ξ_j	Rank
1	0.9906	0.9591	1.3673	1.1946	0.959	0.930	0.945	1
2	0.9089	0.8818	1.3198	1.0000	0.902	0.818	0.860	11
3	0.8338	0.8509	1.1774	1.0456	0.817	0.819	0.818	20
4	0.7681	1.0000	1.0000	1.2354	0.724	0.965	0.844	16
5	1.0000	0.9925	1.2426	1.1836	0.922	0.942	0.932	3
6	0.8514	1.0000	1.0000	1.1836	0.765	0.946	0.856	12
7	0.7886	1.0000	1.0000	1.1836	0.734	0.946	0.840	17
8	0.7301	0.9259	1.0000	1.1836	0.705	0.909	0.807	21
9	0.6754	0.8928	1.0000	1.1461	0.677	0.878	0.778	26
10	1.0000	0.8151	1.4729	1.0204	1.000	0.792	0.896	5
11	0.9384	0.8476	1.3122	1.0204	0.915	0.808	0.861	10
12	0.8809	0.9598	1.2834	1.2314	0.876	0.944	0.910	4
13	0.8259	0.9613	1.1406	1.2968	0.800	0.969	0.885	6
14	0.7153	0.9372	1.0000	1.0680	0.697	0.871	0.784	25
15	0.9841	0.8193	1.2263	1.0350	0.908	0.800	0.854	13
16	0.8101	0.8718	1.1755	1.0517	0.804	0.832	0.818	19
17	0.7462	0.8424	1.1153	1.0707	0.752	0.825	0.788	23
18	0.6913	1.0000	1.0000	1.0707	0.685	0.903	0.794	22
19	0.6532	1.0000	1.0000	1.3144	0.666	0.995	0.831	18
20	1.0000	0.7975	1.3987	1.0000	0.975	0.775	0.875	8
21	1.0000	0.7975	1.3019	1.0158	0.942	0.781	0.862	9
22	0.7936	0.8819	1.1577	1.2569	0.790	0.914	0.852	14
23	0.9019	0.8819	1.0912	1.3154	0.821	0.936	0.879	7

Table. 2. Continued.

DMU_p	$\theta_p^{*opt(1)}$	$\theta_p^{*opt(1)}$	$\theta_p^{*pec(1)}$	$\theta_p^{*pes(1)}$	$\xi_j^{(1)}$	$\xi_j^{(2)}$	ξ_j	Rank
24	1.0000	0.8819	1.0000	1.0848	0.839	0.850	0.845	15
25	0.9301	1.0000	1.2344	1.3271	0.884	1.000	0.942	2
26	0.7922	0.7747	1.1631	1.0376	0.791	0.778	0.785	24
27	0.7440	0.7785	1.0911	1.0510	0.742	0.785	0.764	27

In *Table 2*, the second and third columns indicate the optimistic efficiency scores of the first stage (Leader) and second stage (Follower), all of which are less than or equal to one. A value of 1 indicates that the unit is efficient at that stage. The fourth and fifth columns show the pessimistic efficiency scores for the first stage (Leader) and second stage (Follower), all of which are more than or equal to one. A value of 1 indicates efficiency at that stage. The sixth and seventh columns show the normalized combined efficiency for the first and second stages, calculated with $\Gamma_1 = \Gamma_2 = 0.5$ for DMU_j. The eighth column shows the overall normalized combined efficiency $\lambda = 0.5$. The last column presents the unit rankings, where units 1, 25, and 5 are ranked first, second, and third, respectively.

5 | Conclusion

This study used a standardized two-stage network DEA model to determine the most efficient DMUs. The data set included 27 companies, with AI-generated data to be realistic but not actual values. The network structure consisted of two input variables, two intermediate variables, and one output variable. The Hurwicz criterion is a powerful tool for optimizing the efficiency of two-stage networks in DEA decision-making. This approach enhances resource allocation and precise efficiency evaluation, improving system-wide performance. It helps identify improvement points and optimize resource distribution in complex multi-stage systems. Our research in this article focuses on the simple two-stage network using the VRS model. In the process for further analytic endeavors, efforts could be extended to a more elaborated network, for instance, on a network where part of an initial output is out of the system or an alternative network in the second stage, "new inputs" (Slack variables). We can achieve better results using Hurwicz's approach in the standardized combined efficiency in a two-stage DEA network.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Data Availability

All data generated or analyzed during this study are included in this published article. No additional data are available.

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